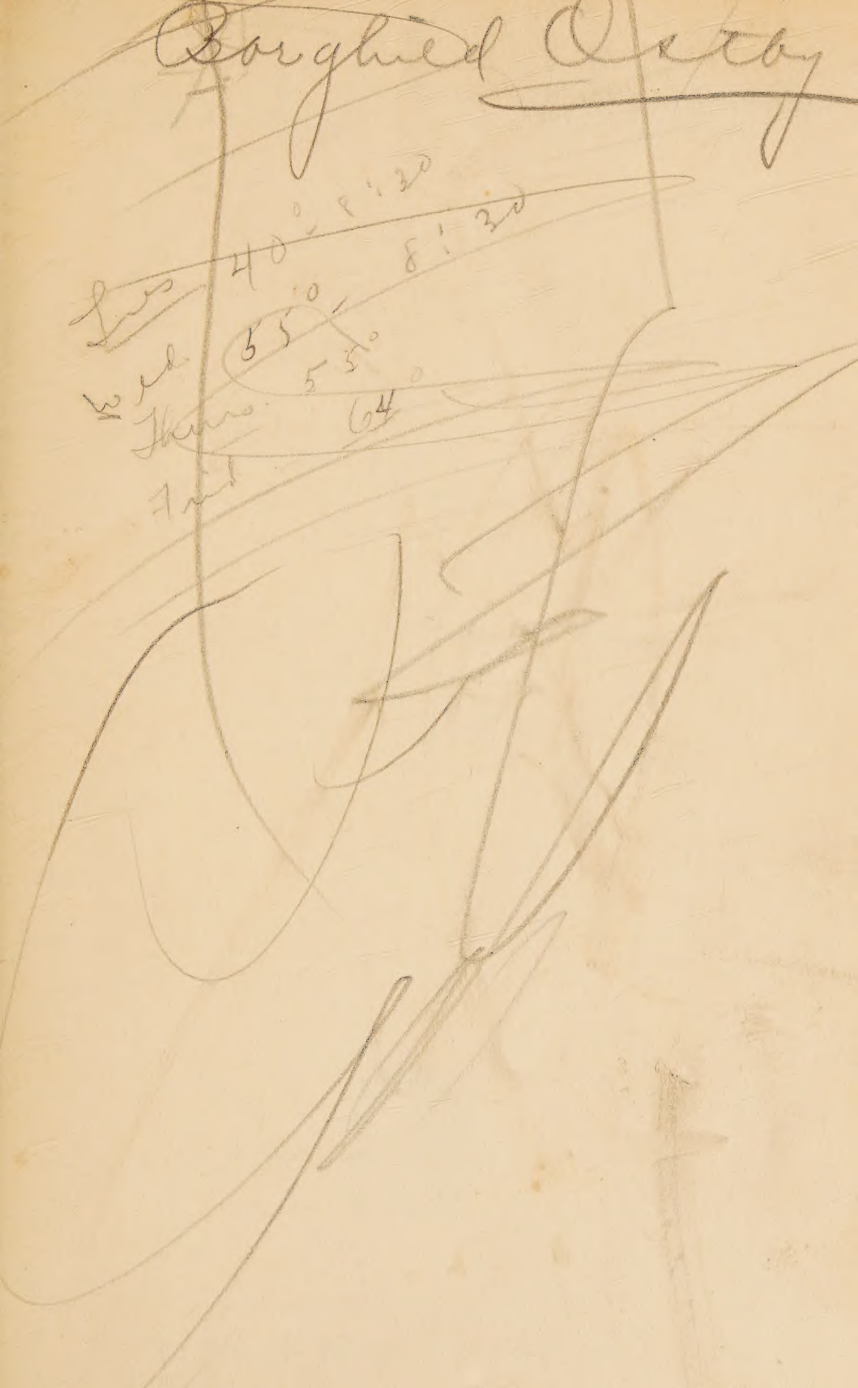
MODERN BUSINESS ARITHMETIC

BRIEF COURSE

FINNEY
AND
BROWN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MODERN BUSINESS ARITHMETIC

BRIEF COURSE

BY

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PREFACE

THIS book presents a short course in the essentials of business arithmetic. Nothing has been included which does not stand the test of business utility. All topics and types of problems which have no application in common business practice have been omitted. The book aims to secure facility in mathematical computation by an abundance of drill, both oral and written, and to secure accuracy by insisting on the checking of results at every point.

Each topic is introduced by a discussion of the business activities related to it. Without sacrificing anything in the way of drill, the material has been selected with a view to giving some notion of the business activities which give rise to the mathematical applications. The problems and exercises are such as actually arise in business; and the methods of solution are those which are employed in business practice. The illustrations and model forms are designed to acquaint the student with actual business methods. The traditional sequence of topics has been abandoned in some cases and those topics have been grouped in which there is a relation of business experience.

The material which has gone into this book has been successfully subjected to the test of preparing hundreds of students to meet the exacting demands of the business world. The ruled forms and the tabulated business statistics so extensively used in the book have demonstrated their effectiveness in developing rapidity, accuracy, and neatness. The authors have consulted many men who are specialists in the various business and industrial activities, and the statements of business customs as well as the tabulation of materials and the arrangement of problems are based on their advice.

The authors wish to acknowledge their indebtedness to the numerous teachers, business men, and professional men who have so kindly aided them by offering valuable suggestions and discriminating criticisms, and by furnishing materials. They wish to acknowledge their especial indebtedness to: Mr. Seymour Walton, Certified Public Accountant, Dean of the Walton School of Commerce, Chicago, who read several of the chapters, the subject matter of which borders on accountancy; Professor Norris A. Brisco, Head of the School of Commerce and of the Departments of Political Economy and Sociology, University of Iowa, Iowa City, Iowa, and Editor of the Efficiency Society Journal; Mr. Stanley C. Crafts, Auditor of Customs, Port of Chicago; Mr. T. H. Fuller, Auditor for Carson, Pirie, Scott & Co., Chicago; Mr. H. A. Brinkman, Cashier of the Harris Trust and Savings Bank, Chicago; and Mr. H. V. Church, Principal of the Cicero Township High School, Berwyn, Ill.

Schools devoting more time to the subject than is required to finish this text are referred to the authors' Complete Course, in which the material of this book is supplemented by further applications to modern business.

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15.	16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.
8	5	3	7	2	5	6	9	6	8	5	7	8	4
6	5	3	8	4	7	4	7	8	5	7	4	8	5
4	6	2	8	3	5	6	5	7	3	6	9	3	6

29.	30.	31.	32.	33.	34.	35.	36.	37.	38.	39.	40.	41.	42.
7	7	4	9	5	7	4	6	8	3	6	5	7	9
6	7	3	6	9	5	8	3	6	7	4	5	7	3
7	4	8	5	3	7	3	8	5	4	2	6	9	4

2. Adding numbers by grouping will increase your speed. Most rapid computers use group addition. For example,

$$\begin{array}{r} 8 \\ 3 \end{array}\Big\}\,11$$
$$\begin{array}{r} 4 \\ 2 \end{array}\Big\}\,6$$
$$17$$

Use this method to find the sums in the following exercise. This work should be done orally.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
6	5	8	4	7	4	8	5	2	6	3	5	4	6
3	2	2	3	4	3	6	7	3	7	7	8	9	6
5	7	3	4	8	5	3	7	5	4	9	4	7	3
4	3	6	2	3	5	7	6	4	8	4	7	3	5

15.	16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.
3	7	4	2	5	3	6	2	4	6	8	9	7	5
3	6	4	8	5	3	7	6	4	8	5	7	3	6
7	5	3	6	5	8	4	5	6	4	7	3	8	2
7	6	3	4	1	7	4	7	4	8	9	5	3	7

29.	30.	31.	32.	33.	34.	35.	36.	37.	38.	39.	40.	41.	42.
3	2	6	8	6	4	2	4	6	4	8	5	2	3
3	6	2	5	8	9	5	3	7	5	3	5	7	9
9	5	3	6	8	9	5	2	6	2	8	5	3	9
5	2	7	4	2	7	3	7	9	4	2	7	4	6

43.	44.	45.	46.	47.	48.	49.	50.	51.	52.	53.
4	8	9	5	3	2	6	8	4	3	6
3	6	8	4	2	6	7	4	3	6	8
7	4	6	3	7	8	5	3	6	7	9
4	7	5	2	8	6	4	8	3	7	9
2	6	4	7	9	6	4	2	6	7	5
7	4	8	3	9	5	9	3	6	7	5

54.	55.	56.	57.	58.	59.	60.	61.	62.	63.
7	3	2	8	5	3	7	3	7	5
5	3	6	8	7	4	2	6	8	3
4	5	6	4	7	3	8	4	2	8
5	2	3	5	7	8	5	3	6	7
4	7	6	4	8	9	2	6	3	7
2	3	6	7	8	4	2	5	6	3

Find the sums in examples 1–14 by the following method:

43
24
67

Think 43, 63, 67.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
24	46	37	53	58	67	48	39	68	73	66	81	93	65
33	52	44	63	47	59	73	58	42	86	59	78	38	75

3. **Recording Addition by Columns.** For convenience in checking, the total of each column may be recorded separately. This will enable you to resume the work where it was discontinued if you are interrupted after having added one or more columns. Three slightly different methods are in use.

a.

369
487
352
896
294
28
37
20
2398

b.

369
487
352
896
294
20
37
28
2398

c.

369
487
352
896
294
28
39
23

Illustrations (*a*) and (*b*) show the method of adding each column *without carrying* the tens. The column totals must be added to obtain the final result. Illustration (*c*) shows the method of adding to each column the amount carried from the column at the right.

Written Work

Make the figures plain and easily legible and place units of the same order in the same vertical column.

Copy and find the sum of:

1.	2.	3.	4.	5.
5823	84239	387935	9328577	38421
4923	42937	512937	4912374	31238
9382	31629	491327	5823746	312
5128	49238	395825	4923748	273845
3258	83153	416239	7239482	39547
7362	42841	482423	1423849	315288
8127	52384	923857	8237421	4234
4239	94856	412831	9423748	21582

6.	7.	8.	9.	10.	11.
27	371	419	372	97	996
83	927	82	496	36	37
95	482	92	18	547	321
84	413	412	739	969	493
82	958	892	847	38	782
15	285	9216	96	437	946
81	381	594	382	692	531
48	294	5	75	99	28
79	148	592	989	831	327

Each student should prepare original examples for addition, following the instruction of the teacher. Exchange papers, criticize the form, and find the sums.

The ability to give and to take dictation of numbers should be developed. Numbers may be dictated by the teacher, or by members of the class.

Written Work

The following table shows the weekly sales report made by a number of salesmen during a certain week.

Find the total sales made by each salesman during the week, and enter these totals in the proper places at the bottom of the table.

Find the total sales made each day, and enter these totals in the column at the right.

Find the total sales made by all salesmen during the week.

The grand total should be found in two ways: by adding the totals at the foot of the blank, and those at the right. If these two grand totals agree, you may assume that the additions are correct; if they do not agree, you should find the error.

DAY	J C OLSEN	D.R.BACON	F.G.BATES	D.O.ESTEY	TOTAL
MONDAY	216 45	538 74	816 98	319 47	
TUESDAY	358 93	619 84	725 48	629 47	
WEDNESDAY	412 57	593 27	639 74	725 84	
THURSDAY	385 13	634 96	724 47	594 72	
FRIDAY	518 92	718 96	639 37	584 38	
SATURDAY	639 47	639 62	729 47	846 94	
TOTAL					

4. Courtis Standards. S. A. Courtis has determined certain standards of achievement in addition, subtraction, multiplication, division, and the copying of figures.

His test in addition includes twenty-four examples similar to the two which follow.

127	996
375	320
953	778
333	886
325	913
911	164
554	897
167	972
554	119

Of the thousands of students examined by means of the Courtis tests about 5% are able to obtain the correct result to twenty-four such examples in addition in eight minutes or less.

NOTE. Information in regard to the Courtis Standards and Tests may be obtained by addressing S. A. Courtis, Detroit, Michigan.

5. How to Rule a Blank Form. Ruling the forms for your written work will furnish you excellent practice in the use of pen, ink, and ruler. This practice will be valuable as a preparation for bookkeeping in school or in a business office. Follow the instructions given below:

a. Make the columns just wide enough for the figures, — about seven figures to an inch. Remember that the total may have one or two more figures than any of the addends (numbers added).

b. Make the horizontal lines parallel and about $\frac{1}{4}$ of an inch apart.

c. Make the spaces for headings and totals $\frac{3}{8}$ of an inch deep.

d. Try to place the form as nearly as possible in the middle of the paper.

6. How to Enter Statistics. Much of the clerk's work in a business office consists of entering statistics on ruled forms.

a. Always write figures on the line.

b. Point off figures in groups of three.

c. Make small figures; they look better and are more easily read than large ones.

d. Keep the figures close to the right side of the column.

e. Keep the columns vertical: units above units, tens above tens, etc.

f. Write the figures at equal distances from each other.

Written Work

1. The following table shows the delivery records of a store running five wagons. A record of each day's deliveries is kept and the week's totals are found. If the summary shows that any one wagon is required to make an unreasonably large number of deliveries, a change in the routes may be necessary. By finding the total number of deliveries made each week, the average cost per delivery may be computed.

DAY	WAGON NO 1	WAGON NO 2	WAGON NO 3	WAGON NO 4	WAGON NO 5	TOTAL
MONDAY	243	319	273	143	241	
TUESDAY	216	326	391	178	216	
WEDNESDAY	257	287	294	121	322	
THURSDAY	301	291	341	147	296	
FRIDAY	294	304	303	218	213	
SATURDAY	318	361	420	223	301	
TOTAL						G.T.

Rule a blank form similar to this model; copy the statistics, and find:

a. The total number of deliveries made during the week by each wagon.

b. The total number of deliveries made each day.

c. The total number of deliveries for the week.

2. Rule a blank form similar to the first of the tables on the opposite page, copy the statistics, and find the totals indicated.

3. Referring to the second of the tables on the opposite page, what was the value of each crop in each of the geographical divisions, and in the entire United States?

Rule a blank in ink and indicate these values as totals.

Weekly Sales Report by Departments

Dept. No.	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Total
1	$932.87	$823.75	$239.75	$724.16	$295.16	$840.19	
2	829.47	584.92	395.58	645.97	396.74	492.10	
3	723.85	486.47	523.78	419.52	296.15	492.58	
4	836.47	594.38	723.91	391.16	749.15	581.07	
5	486.74	432.95	312.85	492.43	723.85	239.57	
6	728.74	483.96	472.58	439.17	396.14	211.64	
7	385.75	539.59	542.87	734.67	723.96	447.92	
8	648.57	823.85	514.29	439.76	824.57	238.11	
9	923.37	239.73	629.85	329.54	385.47	294.10	
10	473.48	865.94	518.59	172.39	749.80	663.19	
Total							Grand Total

Value of the Wheat, Corn, and Oat Crops in the Various States
(In Thousands of Dollars)

State and Division	Corn	Wheat	Oats	State and Division	Corn	Wheat	Oats
Maine	480	72	2,347	Minnesota	28,925	48,938	31,962
New Hampshire	794		225	Iowa	151,207	10,023	58,811
Vermont	1,296	24	1,589	Missouri	112,196	21,375	12,994
Massachusetts	1,629		128	North Dakota	3,766	99,236	20,948
Rhode Island	401		26	South Dakota	28,248	36,008	13,098
Connecticut	2,310		166	Nebraska	67,568	37,985	16,653
New York	13,834	5,306	15,420	Kansas	69,690	68,295	19,264
New Jersey	7,054	1,433	814	N. C. W. Miss. R.			
Pennsylvania	38,797	21,204	14,915				
N. Atlantic				Kentucky	60,192	6,791	1,775
				Tennessee	53,862	7,077	2,632
				Alabama	42,802	359	3,224
				Mississippi	40,356	93	1,180
Delaware	3,381	1,864	55	Louisiana	22,093		361
Maryland	13,450	8,536	608	Texas	98,112	10,253	13,390
Virginia	33,739	8,682	2,020	Oklahoma	41,770	15,072	7,988
West Virginia	15,928	3,412	1,461	Arkansas	33,828	884	1,741
N. Carolina	42,418	5,907	2,352	S. Central			
S. Carolina	29,136	865	4,598				
Georgia	45,864	1,498	4,921	Montana	428	12,381	7,997
Florida	6,727		518	Wyoming	236	1,745	3,171
S. Atlantic				Colorado	4,368	8,006	4,717
				New Mexico	1,562	1,109	828
				Arizona	528	778	188
				Utah	202	4,544	2,069
				Nevada	29	1,137	208
Ohio	78,484	9,565	30,782	Idaho	276	9,613	5,956
Indiana	83,733	9,374	23,940	Washington	651	36,535	5,476
Illinois	174,791	8,641	54,818	Oregon	472	15,132	5,623
Michigan	31,492	6,720	17,103	California	1,635	5,850	4,290
Wisconsin	29,714	2,958	27,119	Far Western			
N. C. E. Miss. R.				United States			

Note. Different classes, or different groups of students of the same class, should be held responsible for computing the desired sums for different geographical divisions.

Checking Addition by Casting Out Nines

7. Excess of Nines. The remainder after dividing any number by nine is called the *Excess of Nines.* If the number 47 is divided by 9, the excess of nines is 2. The excess of nines in 30 is 3, because when 30 is divided by 9, there is a remainder of 3. When we "cast out nines" from a number, we divide the number by 9, and indicate the excess.

What is the excess of nines in 17? 14? 39? 45? 117? 23?

8. Method of Checking. If two numbers, each exactly divisible by 9, are added, their sum also is divisible by 9. If one number with an excess of 3 is added to a number with an excess of 2, the sum will have an excess of 5. *The excess of nines in a sum is equal to the excess in the sum of the excesses of the numbers added.*

Examples. Add 30 and 19; check the result by casting out nines.

Solution: $30 + 19 = 49$.
The excess of nines in the sum, 49, is 4.
30 has an excess of 3;
19 has an excess of 1.
The sum of the excesses of the numbers added is 4, therefore the addition checks.

For convenience, the work may be arranged as follows:

	Numbers Added	Excesses
a.	30	3
	19	1
Sum	49	4
Excess of sum	4	

	Numbers Added	Excesses
b.	23	5
	13	4
Sum	36	9 or 0
Excess of sum	0	

	Numbers Added	Excesses
c.	49	4
	34	7
Sum	83	11 or 2
Excess of sum	2	

9. Short Method of Finding the Excess. The excess of nines in any number may be found by the following method :

Add the digits composing the number. If the sum is composed of two or more digits add them. Continue this procedure until a result of one digit is secured.

Examples. 1. What is the excess of nines in 25 ?

SOLUTION: $2 + 5 = 7$, the excess.

2. What is the excess of nines in 328 ?

SOLUTION: $3 + 2 + 8 = 13$
$1 + 3 = 4$, the excess.

The addition of digits to find the excess will be further simplified by observing the following suggestions :

a. Ignore 9's, and combinations which add to 9 or multiples of 9.

Example. What is the excess of nines in 9457 ?

SOLUTION: Ignore 9 and $5 + 4$; the excess is 7.

b. When the addition results in a number of two digits, add the digits and proceed as before.

Example. What is the excess of nines in 7528 ?

SOLUTION: $7 + 5 = 12$; $1 + 2 = 3$;
$3 + 2 + 8 = 13$; $1 + 3 = 4$, the excess.

10. Limitation of the Method. Casting out nines is not an absolute check for any process. It will not disclose an error of 9 or any multiple of nine, neither will it disclose an interchange of digits, such as 763 for 673.

Written Work

1. Add and check :

Numbers to be Added	Excesses
213	?
412	?
617	?
362	?
145	?
215	?
829	?
657	?
923	?
?	?

2. Rule a suitable blank form for the following : At the right of each column of figures rule a narrow column in which to indicate the excesses.

Find the totals as indicated ; check all additions by casting out nines.

Value of Farm Lands, Buildings, and Implements in the United States

Division	Land	Ex.	Buildings	Ex.	Implements	Ex.	Total	Ex.
New England .	382,134,424		336,410,384		50,798,826			
Middle Atlantic	1,462,321,005		980,628,098		167,480,384			
E. N. Central .	7,231,699,114		1,642,292,480		268,806,550			
W. N. Central .	10,052,560,913		1,562,104,957		368,635,544			
South Atlantic .	1,883,349,675		603,086,799		98,230,147			
E. S. Central .	1,326,826,864		411,570,975		75,339,333			
W. S. Central .	2,716,098,530		412,498,352		119,720,377			
Mountain. . .	1,174,370,096		145,026,777		49,429,975			
Pacific	2,246,313,548		231,832,706		66,408,647			
U. S. Totals .							Grand Total	

3. The following table shows a method used by department stores to determine the cost of the purchases made for various departments during the month. For example, Invoice No. 1,

purchased on January 2, included goods for the six departments of the store. Invoice No. 2 included goods for only three departments.

The total amount of the invoice may be found by horizontal addition.

The total purchases for each department during the month may be found by vertical addition.

Rule a blank suitable for this material, enter the data, and find:

a. The total of each invoice.

b. The total purchases for each department during the month.

c. The total purchases for all departments during the month.

Check all additions by casting out nines.

Invoice No.	Date	Dept. 1	Dept. 2	Dept. 3	Dept. 4	Dept. 5	Dept. 6	Total
1	Jan. 2	129 30	340 60	290 39	360 89	240 30	376 80	
2	Jan. 3	276 90		347 80			174 44	
3	Jan. 3		295 67	298 87	350 27	289 67	287 57	
4	Jan. 5	246 32	238 47	350 27	347 25	250 68	276 57	
5	Jan. 6	231 53	350 21		378 89		190 23	
6	Jan. 8	212 23	325 67	345 67	367 78	310 57	185 67	
7	Jan. 8	227 85		346 78	357 89	298 67		
8	Jan. 10						170 67	
9	Jan. 11	160 87	325 67	287 67	367 57	287 00	189 00	
10	Jan. 15	170 97	341 28	321 67	378 98	287 89		
11	Jan. 15	180 89		333 33		278 90	234 86	
12	Jan. 15		345 67	345 67	328 98	300 00	350 90	
13	Jan. 16	203 25	356 78		378 68			
14	Jan. 17	204 67		367 78	376 56	267 87	347 52	
15	Jan. 19		290 87	356 87		278 65	378 67	
16	Jan. 23		297 76	347 89	398 67	256 28	250 00	
17	Jan. 24	234 56			388 88			
18	Jan. 25	245 67	287 98	367 34		256 78	234 56	
19	Jan. 28		229 39	323 23	376 29	234 56	167 89	
20	Jan. 31	236 78	310 21	289 98	387 85	245 67	245 67	
Total								

CHAPTER II

SUBTRACTION

11. Drill Tables. Daily practice on the following combinations will increase your speed and accuracy in the process of subtraction.

Drill on these combinations until you can state all of the results in less than thirty seconds.

9	8	7	9	6	7	4	7	3	6	8	7	8	4	5	8
2	1	5	3	1	2	4	4	3	6	4	3	8	2	1	3
2	7	9	5	9	5	6	5	9	4	4	9	8	2	6	8
1	7	4	5	5	4	3	2	7	4	3	8	5	2	4	6
5	8	4	8	6	7	7	9	8	1	3	3	5	7	9	6
3	8	1	6	2	1	6	1	2	1	2	1	5	7	6	5

Drill on the following until you can state all of the results in less than thirty seconds.

16	14	15	18	13	17	12	11	16	11	12	17	11
8	6	8	9	4	9	5	9	7	6	6	8	4
13	12	15	16	11	12	12	12	11	13	15	11	12
5	3	6	9	3	3	8	7	7	9	7	8	9
11	12	13	14	14	13	14	15	14	11	13		
5	4	7	8	7	6	5	9	9	2	8		

When performing subtraction, one is frequently obliged to "borrow" as in the following.

Copy these examples, and make the subtractions.

1.	2.	3.	4.	5.	6.
479	536	4279	14973	3084	12379
284	198	3682	8297	1596	3084

7.	8.	9.	10.	11.
42937	946320	37649	947302	794680
39048	298452	20863	842943	409694

12. Checking Subtraction. Subtraction may be checked by either of the following methods:

(a) Difference + Subtrahend = Minuend.

(b) Minuend − Difference = Subtrahend.

Example. Subtract 21 from 78.

Solution.	78	Minuend	Check.	a.	57	Difference	b.	78	Minuend
	21	Subtrahend			21	Subtrahend		57	Difference
	57	Difference			78	Minuend		21	Subtrahend

Written Work

Perform the subtractions indicated; check the first five by method (a), and the last five by method (b),

1.	2.	3.	4.	5.
489	7932	82578	63217	49216
392	494	58214	41283	21953

6.	7.	8.	9.	10.
58310	40513	53821	79608	184732
49168	39458	41927	20973	75806

11. The following data show the sales in the various departments of a large store for the month of August, 1914, and the month of August, 1915.

Rule a blank similar to the following:

COMPARATIVE SALES RECORD

DEPARTMENT NO.	AUGUST, 1914	AUGUST, 1915	INCREASE OR DECREASE
1	$ 5,483.85	$ 5,834.75	
2	7,239.74	8,238.74	
3	15,493.67	14,835.92	
4	9,834.56	10,385.47	
5	23,842.12	22,147.94	
6	15,858.43	16,614.34	
7	6,395.75	6,114.32	
8	19,432.86	22,324.73	
9	9,445.87	10,302.95	
10	43,221.42	46,932.56	
11	29,496.56	31,416.47	
12	18,853.16	17,542.85	
13	22,542.19	25,436.83	
14	25,746.91	24,427.85	
15	18,422.16	21,541.86	
Total			

Enter the sales in the proper column of the blank.

Find the increase or decrease in the amount of the sales in each department for August, 1915, over the sales in the same department for August, 1914.

Enter increases in black ink; decreases in red ink.

Find the net increase for the entire store.

12. The following shows the total population of the United States for the years indicated.

1790—3,929,214; 1800— 5,308,483; 1810— 7,239,881;
1820—9,638,453; 1830—12,866,020; 1840—17,069,453;
1850—23,191,876; 1860—31,443,321; 1870—38,558,371;
1880—50,155,783; 1890—62,947,714; 1900—75,994,575;
1910—91,972,266.

Rule a blank form to show the year the census was taken, the population each census year, and the increase in population for each interval of 10 years.

See how simple you can make this blank.

13. Rule a form, enter the following statistics, find the gross profit of each department, and the gross profit of the entire store.

GROSS PROFIT OF A DEPARTMENT STORE

DEPT. NO.	SALES	COST OF GOODS SOLD	GROSS PROFIT
1	$5,629.80	$4,984.37	
2	7,358.92	6,295.16	
3	4,916.09	4,192.86	
4	7,329.16	6,593.54	
5	10,609.15	9,835.17	
6	6,123.18	5,902.52	
7	7,212.47	6,275.43	
8	9,475.37	8,594.48	
9	4,238.16	3,725.91	
10	5,824.78	5,014.78	
Total			

14. Rule a suitable form, enter the following statistics, and find :

a. The gross profit by departments.

b. The gross profit of the entire store (two ways).

c. The net profit or net loss by departments; enter net profit in black, net loss in red.

d. Net profit or net loss of the entire store (two ways).

DEPT. NO.	SALES	COST OF GOODS SOLD	GROSS PROFIT	EXPENSES	NET PROFIT OR LOSS
1	$7816.40	$6715.32		$423.85	
2	9317.42	8492.17		530.08	
3	6842.56	5914.72		731.96	
4	7319.62	6593.57		671.42	
5	8295.17	7942.17		456.72	
6	5732.88	5101.59		322.75	
7	9514.82	8899.55		693.15	
8	2289.74	1856.80		216.77	
9	5793.66	5135.60		788.51	
10	9559.38	8625.50		523.80	

13. Subtracting by Adding Complements. A series of additions and subtractions may be performed by the method of adding complements.

The difference between any number and the next higher power of 10 is called the *complement* of the number. Thus, the complement of 7 is 3; the complement of 6 is 4; the complement of 83 is 17.

If, instead of subtracting a number less than ten from a given number, its complement be added, the result will be 10 too large. Thus, $13 - 6 = 7$ or $13 + 4 = 17$ (a result 10 too large).

If, instead of subtracting *two* numbers less than ten from a given number, their complements are added, the result will be 20 too large; if the complements of three such numbers are added, the result will be 30 too large, etc.

Examples. 1. $29 - 6 - 7 = ?$

SOLUTION. $29 + 4 + 3 = 36$. Since two complements were added, the result is 20 too large. Therefore, subtract 20, leaving 16.

2. $38 - 4 - 7 - 8 = ?$

SOLUTION. $38 + 6 + 3 + 2 = 49$. Since three complements were added, subtract 30, leaving 19.

3. $46 - 8 + 4 - 7 = ?$

SOLUTION. $46 + 2 + 1 + 3 = 55$. Since two complements were added, 20 must be subtracted, leaving 35.

The practical value of this method will be shown by solving the following examples.

1. $36 - 24 + 19 - 12 + 21 - 13 - 6 = ?$

SOLUTION. By combining as indicated in the units' column, beginning at the top (complements are marked with an *) we have

$$6 + 6^* + 9 + 8^* + 1 + 7^* + 4^* = 41$$

Since four complements were added, the result is 40 too large. Therefore write 1 and drop the 4.

By combining as indicated in the tens' column we have

$$3 + 8^* + 1 + 9^* + 2 + 9^* = 32$$

Since three complements were added, the result is 30 too large. Write 2 and drop the 3.

Result, 21.

36
− 24
19
− 12
21
− 13
− 6
21

2. $985 + 234 - 126 - 34 - 125 - 174 + 386 = ?$

$$\begin{array}{r} 985 \\ 234 \\ -\ 126 \\ -\ 34 \\ -\ 125 \\ -\ 174 \\ \underline{386} \\ 1146 \end{array}$$

Solution. Combining in the units' column, beginning at the top we have

$$5 + 4 + 4^* + 6^* + 5^* + 6^* + 6 = 36$$

Write the 6. Since four complements were added, the result is 40 too large. Therefore we must drop the 3 in 36, and also subtract 1 from the 8 at the top of the tens' column.

Combining in the tens' column we have

$$7 + 3 + 8^* + 7^* + 8^* + 3^* + 8 = 44$$

Since four complements were added, we must deduct 40, leaving 4 which is written in the tens' column of the result.

Combining in the hundreds' column we have

$$9 + 2 + 9^* + 9^* + 9^* + 3 = 41$$

Three complements were added, the result is therefore 30 too large. Subtract 30 and write 11.

Result, 1146.

Oral Work

Find the results by the addition of complements:

1. $46 - 4 - 7 =$
2. $72 - 8 - 6 - 3 =$
3. $39 - 7 + 3 - 9 =$
4. $48 - 17 - 6 =$
5. $81 - 23 - 42 =$
6. $49 - 22 + 15 - 11 =$
7. $85 - 26 - 35 + 62 - 16 =$
8. $643 - 289 + 364 =$
9. $781 - 247 + 64 =$
10. $1046 - 987 + 649 =$
11. $943 - 876 + 629 =$
12. $1349 - 268 + 421 =$

Written Work

1.	2.	3.
2346	1932	923,578
+ 1267	+ 9845	− 5,284
− 321	− 932	+ 28,956
+ 6964	+ 2122	− 123,749
− 1235	− 2375	+ 83,219
+ 367	− 23	− 259,734
− 2960	+ 692	+ 125,982
− 985	− 1243	− 429,764

Written Work

1. The following model shows the ruling of the ledger in which banks keep accounts with their depositors. Deposits are added to the balance of the previous day, and checks are subtracted, to find the new balance to the credit of the depositor's account.

Find the daily balances:

WILLIAM HATFIELD

DATE	DEPOSITS	CHECKS	BALANCE
June 1			$328.57
	$ 85.68	$ 14.25	?
2	125.80	65.90	
		12.73	?
3	245.85	127.89	
		114.56	
		35.87	?
4	319.45	38.51	
	26.82	102.50	?
5		95.90	
		141.66	
		95.68	?
6	450.00	132.88	
		6.20	?
8	139.75	216.45	
		39.46	
		19.99	
		213.55	
		66.82	?

2. The following blank shows a convenient method of keeping a record of cash received and paid by a small business.

Rule a blank similar to the model and enter the statistics.

DATE	CASH RECEIVED		CASH PAID			DAILY CASH
	Cash Sales	On Account	Purchases	On Account	Expenses	Balance
Feb. 1						$216.27
1	$215.78	$124.35	$ 80.75	$ 68.55	$ 8.75	?
2	294.80	95.88	157.47	113.25	3.73	?
3	188.47	148.23	95.48	316.57	27.49	?
4	218.43	49.47	17.49	42.39	9.12	?
5	388.92	112.67	221.81	76.29	75.84	?
6	478.07	290.04	88.12	448.14	6.90	?
	?	?	?	?	?	?

a. Find the daily cash balances.

To the balance of the preceding day add the receipts from cash sales and receipts on account, and subtract the various amounts listed under "Cash Paid."

Thus, $\$216.27 + 215.78 + 124.35 - 80.75 - 68.55 - 8.75 = ?$

b. Find the totals for each column.

3. Add upward; subtract across:

a.		*b.*	
7,463	2,847 =	174,638	94,273 =
5,928	3,804 =	38,270	27,409 =
9,604	2,870 =	70,563	2,879 =
3,962	1,436 =	924,360	14,287 =
9,287	5,426 =	7,503	2,769 =
	=		=

CHAPTER III

MULTIPLICATION

ACCURACY and speed in multiplication depend largely upon a thorough mastery of the multiplication tables. The student should thoroughly review the tables previously learned and should continue with daily drills on combinations up to 25 times 25.

14. Drill Tables.

Multiply across ; add upward :

1.	2.	3.	4.
$74 \times 436 =$	$83 \times 423 =$	$25 \times 624 =$	$35 \times 624 =$
$74 \times 523 =$	$83 \times 157 =$	$25 \times 726 =$	$35 \times 706 =$
$74 \times 287 =$	$83 \times 284 =$	$25 \times 37 =$	$35 \times 753 =$
$74 \times 492 =$	$83 \times 307 =$	$25 \times 869 =$	$35 \times 496 =$
$74 \times ? =$	$83 \times 596 =$	$25 \times 493 =$	$35 \times 548 =$
	$83 \times ? =$	$25 \times 468 =$	$35 \times 784 =$
		$25 \times ? =$	$35 \times ? =$

Oral Work

Use 2, 3, 4, 5, 6, 7, 8, and 9 as multipliers. Name the results for each column in less than 20 seconds.

1.	2.	3.	4.	5.
3	18	5	4	9
5	10	8	9	11
7	4	12	15	16
14	7	15	20	30
11	9	7	17	25
6	15	30	8	14
8	16	22	13	17
9	20	16	18	11
15	12	25	22	16

Written Work

1. A factory made an investigation of the number of articles of a certain kind manufactured by each of its employees. It found that:

9 men produced 46 articles each.
9 men produced 48 articles each.
11 men produced 53 articles each.
15 men produced 55 articles each.
18 men produced 59 articles each.
23 men produced 61 articles each.
25 men produced 62 articles each.
25 men produced 63 articles each.
21 men produced 64 articles each.
17 men produced 65 articles each.
17 men produced 66 articles each.
14 men produced 68 articles each.
12 men produced 69 articles each.
8 men produced 70 articles each.

Rule a form with a heading similar to the following:

PRODUCTION RECORD

NUMBER OF MEN	NUMBER OF ARTICLES MADE BY EACH	TOTAL

Enter the statistics, find the number of articles produced by each group of employees, and the total number of articles produced by all of the employees.

2. Nine workmen were employed in the manufacture of different articles, and were paid a certain number of cents for each piece completed. Complete the following table, finding the wages earned by each workman.

DAILY PIECEWORK LABOR COST

WORKMAN NO.	NUMBER OF ARTICLES MADE	WAGE RATE PER PIECE	WAGES EARNED
1	17	$ 0.27	
2	19	.22	
3	38	.16	
4	79	.06	
5	32	.15	
6	28	.18	
7	64	.07	
8	81	.05	
9	29	.21	

15. Checking Multiplication. Multiplication may be checked by several methods. The following methods are commonly used.

a. Repeating the multiplication and assuming that if the same product is obtained the work is correct. This is not a reliable check because an error may be repeated.

b. Dividing the product by the multiplier to obtain the multiplicand, or by the multiplicand to obtain the multiplier.

c. Casting out nines.

16. Casting Out Nines. The method is as follows: *Find the excess of nines in the multiplicand and in the multiplier. Find the product of these excesses. Find the excess of nines in this product. It should equal the excess of nines in the result.*

Example. Multiply 23 by 16; check by casting out nines.

SOLUTION. 23 — 5 = the excess of nines in 23.
16 — 7 = the excess of nines in 16.
368 — 35 = the product of these excesses.
CHECK. — 8 = the excess in this product.
The excess of nines in 368 is also 8. The multiplication, therefore, checks.

Without performing the multiplications determine the probable correctness of the following products, by means of casting out nines:

1.	2.	3.	4.	5.	6.	7.
25	82	36	286	4172	344	733
38	35	87	37	39	281	492
950	2870	3132	10,382	162,698	96,664	362,636

8.	9.	10.
398	43,962	34,276
241	47,835	21,578
95,918	2,102,922,270	738,627,528

Written Work

Multiply and check by casting out nines :

11.	12.	13.	14.	15.	16.
347	279	627	132,879	63,154	78,293,567
861	439	123	642,378	9,837	20,417,839

17. The following table gives information regarding the corn crop in the United States in a recent year.

CORN PRODUCTION IN THE UNITED STATES

STATES	NUMBER OF THOUSANDS OF ACRES	AVERAGE YIELD PER ACRE	AVERAGE PRICE PER BUSHEL IN CENTS
North Atlantic			
Maine	16	40.0	75
New Hampshire	23	46.0	75
Vermont	45	40.0	75
Massachusetts	47	45.0	77
Rhode Island	11	41.5	86
Connecticut	60	50.0	77
New York	512	38.6	70
New Jersey	273	38.0	68
Pennsylvania	1,499	42.5	63
South Atlantic			
Delaware	195	34.0	51
Maryland	670	36.5	55
Virginia	1,980	24.0	71
West Virginia	725	33.8	65
North Carolina	2,808	18.2	83
South Carolina	1,915	17.9	85
Georgia	3,910	13.8	85
Florida	655	13.0	79
North Central, East of Miss.			
Ohio	4,075	42.8	45
Indiana	4,947	40.3	42
Illinois	10,658	40.0	41
Michigan	1,625	34.0	57
Wisconsin	1,632	35.7	51

CORN PRODUCTION IN THE UNITED STATES — *Continued*

STATES	NUMBER OF THOUSANDS OF ACRES	AVERAGE YIELD PER ACRE	AVERAGE PRICE PER BUSHEL IN CENTS
North Central, West of Miss.			
Minnesota	2,266	34.5	37
Iowa	10,047	43.0	37
Missouri	7,622	32.0	46
North Dakota	328	26.7	43
South Dakota	2,495	30.6	37
Nebraska	7,609	24.0	31
Kansas	7,575	23.0	40
South Central			
Kentucky	3,600	30.4	55
Tennessee	3,332	26.5	61
Alabama	3,150	17.2	79
Mississippi	3,106	18.3	71
Louisiana	1,805	18.0	68
Texas	7,300	21.0	64
Oklahoma	5,448	18.7	41
Arkansas	2,475	20.4	67
Far Western			
Montana	24	25.5	70
Wyoming	16	23.0	64
Colorado	420	20.8	50
New Mexico	93	22.4	75
Arizona	19	33.0	100
Utah	9	30.0	75
Nevada	1	30.0	97
Idaho	12	32.8	70
Washington	31	27.3	77
Oregon	20	31.5	75
California	52	37.0	81

Prepare a blank similar to the model.

STATES	ACRES		AVERAGE YIELD PER ACRE		TOTAL BUSHELS PRODUCED		AVERAGE PRICE PER BUSHEL		TOTAL CROP VALUE		
		Ex.		Ex.		Ex.		Ex.	$	¢	Ex.

a. Find the total number of bushels of corn produced in each state.

b. Find the total value of the corn crop of each state.

Different classes or various groups of a class should make the required computations for assigned geographical divisions of the country. In checking the results, the "excesses" should be placed in the columns marked "Ex."

Many interesting comparisons may be made from the data of the preceding table. For illustration: Name the five states which produced the most corn and compare the yields in these states. What geographical section of the country produced the largest corn crop? What relation, if any, is there between the size of the corn crop in the various states and the average price per bushel in the various states?

Short Methods of Multiplication

These short methods will be found to be very practical. Master two or three of them thoroughly before taking up others. Use those that you have mastered whenever you have opportunity to do so. It is not necessary that all of these short methods be mastered before the succeeding chapters are studied.

17. To multiply by 10, 100, 1000, 10000, etc.

a. When the multiplicand is an integer. *Annex to the multiplicand as many zeros as there are zeros in the multiplier.*

Thus, to multiply an integer by 10, annex one zero; to multiply by 100, annex two zeros.

Examples. 1. $37 \times 10 = 370$. 2. $29 \times 100 = 2900$.

b. When the multiplicand is a decimal fraction. *Move the decimal point as many places to the right as there are zeros in the multiplier.*

It may be necessary to annex zeros in order to move the decimal point the desired number of places.

Examples. 1. Multiply .1357 by 100.

Solution. Move the decimal point two places to the right, 13.57.

2. Multiply 32.46 by 1000.

Solution. In order to move the decimal point three places to the right, it is necessary to annex one zero, giving, as a result, 32,460.

Oral Work

Multiply as indicated :

1. $37{,}946 \times 100$.
2. $5293 \times 10{,}000$.
3. $639 \times 100{,}000$.
4. 120×1000.
5. $.376 \times 1000$.
6. 1.349×1000.
7. 27.9637×100.
8. $.000932 \div 10$.
9. $.00873 \times 1000$.
10. $.7032 \times 1000$.
11. 3.69×1000.
12. $.0027 \times 100{,}000$.
13. $1427.834 \times 10{,}000$.
14. 625.086×1000.

18. To multiply numbers ending with zeros.

a. When both numbers are integers. *Multiply the numbers represented by the significant figures. To the product thus obtained, annex as many zeros as there are final zeros in both the multiplicand and multiplier.*

Example. Multiply 3400 by 1200.

SOLUTION. $34 \times 12 = 408$. Annexing four zeros, we obtain the product 4,080,000.

Perform the following multiplications. Whenever possible, do the work orally.

1. 169×300.
2. 210×300.
3. 4567×700.
4. 1390×1200.
5. 2300×1500.
6. $19{,}000 \times 16$.
7. $3194 \times 23{,}000$.
8. 420×3400.

b. When one of the numbers is a decimal fraction. *Multiply the numbers represented by the significant figures. Move the decimal point as many places to the right in the product as there are final zeros in the integer.* (This may necessitate annexing zeros.)

Examples.

1. Multiply .486 by 300.

SOLUTION. $3 \times .486 = 1.458$.

Move the decimal point two places to the right, the result is 145.8.

2. Multiply 3.2 by 400.

SOLUTION. $4 \times 3.2 = 12.8$.

Move the decimal point two places to the right, the result is 1280.

Written Work

Perform the following multiplications:

1. $.47 \times 200$.
2. 3.786×4000.
3. 17.682×500.
4. $.0746 \times 3000$.
5. $.072 \times 6000$.
6. $.382 \times 1200$.
7. $.0837 \times 14000$.
8. $.042 \times 170$.
9. $.0036 \times 1500$.
10. 4.26×7000.

19. To multiply by 9, 99, 999, etc.

a. To multiply by 9. *Annex one zero to the number to be multiplied, thus multiplying it by* 10; *from this result subtract the number to be multiplied.*

Example. Multiply 346 by 9.

SOLUTION.

$$\begin{array}{r} 3460 \\ 346 \\ \hline 3114 \end{array}$$

b. To multiply by 99. *Annex two zeros to the number to be multiplied, thus multiplying it by* 100; *from this result subtract the number to be multiplied.*

Example. Multiply 293 by 99.

SOLUTION.

$$\begin{array}{r} 29300 \\ 293 \\ \hline 29007 \end{array}$$

Written Work

Multiply each of the following numbers by 9, 99, and 999:

1. 632. 2. 748. 3. 135. 4. 737. 5. 427. 6. 166.

20. To multiply by numbers slightly smaller than 10, 100, 1000, 10,000, etc.

A modification of the short method explained in the preceding section may be used to multiply by numbers slightly smaller than 10, 100, 1000, 10,000, etc.

To multiply by 98. *Annex two zeros to the number to be multiplied and from this result subtract twice the number to be multiplied.*

How can the short method be used if you are to multiply by 97, 96, 95, or 998?

Written Work

Apply short methods to the following examples:

1. 675×96.	**2.** 350×95.	**3.** 535×91.
4. 687×97.	**5.** 94×3.4.	**6.** 995×82.
7. 634×994.	**8.** $.48 \times 997$.	**9.** 23×99.
10. 48×997.	**11.** 12×988.	**12.** 47×998.

21. To multiply by 11.

a. When the multiplicand contains two digits.

Place between these two digits, their sum.

Example. $34 \times 11 = 374$.

When the sum of the two digits is 10 or more, 1 must be carried to the digit at the left.

Example. $68 \times 11 = 748$.

Written Work

Multiply each of the following numbers by 11:

1. 27.	**2.** 63.	**3.** 93.	**4.** 74.	**5.** 26.	**6.** 35.
7. 22.	**8.** 87.	**9.** 28.	**10.** 46.	**11.** 75.	**12.** 96.
13. 37.	**14.** 36.	**15.** 57.	**16.** 85.	**17.** 98.	**18.** 72.

b. When the multiplicand contains three or more digits:

The units' digit of the multiplicand is the units' digit of the product; the sum of the units' and tens' digits is the tens' digit of the product; the sum of the tens' and hundreds' digits is the hundreds' digit of the product, etc.

Whenever the sum of two digits is ten or more, 1 must be carried.

Examples. 1. Multiply 793 by 11.

SOLUTION. 3 (the units' digit of the multiplicand) becomes the units' digit of the product.

$9 + 3 = 12$ (carry the 1)

$1 + 7 + 9 = 17$ (carry the 1)

$1 + 7 = 8$

8723

2. Multiply 52,635 by 11.

SOLUTION. $52{,}635 \times 11 = 578{,}985$.

Written Work

Multiply each of the following by 11:

1. 363.
2. 271.
3. 823.
4. 456.
5. 3742.
6. 876,394.
7. 3,578,962.
8. 34,579.
9. 263,789.
10. 123,496,287.

22. To multiply by 111.

$362{,}941 \times 111 = ?$

When the multiplication is performed in the customary manner, the multiplicand is repeated as follows:

362941
362941
362941

When the short method is applied, *the units' digit of the multiplicand is the units' digit of the product; the sum of the units' and tens' digits of the multiplicand is the tens' digit of the product; the sum of the units', tens', and hundreds' digits of the multiplicand is the hundreds' digit of the product; the sum of the tens', hundreds', and thousands' digits of the multiplicand is the thousands' digit of the product, etc.*

The excess above 10 is always to be carried to the next sum.

Written Work

Without recopying, write the products obtained by multiplying each of the following by 111:

1. 729,361.
2. 124,396.
3. 1,793,862.
4. 5,374.
5. 235,692.
6. 8,354,927.

23. To multiply two numbers ending in 5.

a. When the sum of the digits at the left of the 5's is an even number.

Multiply the digits at the left of the 5's; to this product add one half the sum of these digits; to this result annex 25.

Examples. 1. Multiply 65 by 25.

SOLUTION. $2 \times 6 = 12$, the product of the digits at the left of the 5's.

$\frac{1}{2}$ of $(2 + 6) = 4$

16

1625, result obtained by annexing 25.

2. Multiply 625 by 445.

SOLUTION. $62 \times 44 = 2728$

$\frac{1}{2}$ of $(62 + 44) = 53$

2781

278125, result obtained by annexing 25.

Written Work

Multiply as indicated:

1. 35×75.
2. 95×75.
3. 35×55.
4. 25×45.
5. 325×45.
6. 725×65.
7. 835×175.
8. 145×165.
9. 195×115.
10. 225×185.

b. When the sum of the digits at the left of the 5's is an odd number.

Multiply the digits at the left of the 5's; to this product add one half the sum of these digits, dropping the $\frac{1}{2}$; to this result annex 75.

Examples. 1. Multiply 75 by 45.

SOLUTION. $4 \times 7 = 28$, the product of the digits at the left of the 5's.

$\frac{1}{2}$ of $(4 + 7) = \underline{5\frac{1}{2}}$

$33\frac{1}{2}$

3375, the result obtained by dropping the fraction $\frac{1}{2}$, and annexing 75.

2. Multiply 325 by 475.

SOLUTION. $32 \times 47 = 1504$

$\frac{1}{2}$ of $(32 + 47) = \underline{39\frac{1}{2}}$,

1543, adding and dropping the $\frac{1}{2}$.

154,375, result secured by annexing 75.

Written Work

Multiply as indicated:

1. 75×65.
2. 125×135.
3. 95×85.
4. 145×175.
5. 225×75.
6. 165×135.
7. 145×215.
8. 435×125.

24. To multiply two numbers, when certain digits of the multiplier are contained an integral number of times in other digits of the multiplier.

Multiply 224 by 279.

Since 9 is contained 3 times in 27, first multiply 224 by 9, then multiply this product by 3.

Thus,

$$\begin{array}{r} 224 \\ 279 \\ \hline 2016 = 9 \times 224 \\ 6048 = 3 \times 2016 \\ \hline 62496 \end{array}$$

When using this method, be careful to place the product of the second multiplication in the proper position.

Example. 1. Multiply 341 by 618.

$$\begin{array}{r} 341 \\ 618 \\ \hline 2046 = 6 \times 341 \\ 6138 = 3 \times 2046 \\ \hline 210738 \end{array}$$

SOLUTION. Since 18 is a multiple of 6, multiply first by 6. Place the right-hand figure of the product, 2046, directly under the 6 of the multiplier. The product of 18×341 can now be obtained by multiplying 2046 by 3. The right-hand figure of this product, 6138, is placed directly under the 8 of the multiplier.

Written Work

Multiply the following, stating by what numbers you multiplied in order to take advantage of the short method:

1. 468×243.	2. $1{,}235 \times 981$.	3. 719×427.
4. 687×654.	5. 739×848.	6. $7{,}362 \times 1{,}248$.
7. $1{,}247 \times 1{,}864$.	8. $146{,}387 \times 315$.	9. $1{,}235 \times 819$.

25. The **supplement** of a number is the difference between the number and the next lower power of 10. The supplement of 15 is 5; the supplement of 134 is 34; the supplement of 1042 is 42.

26. To multiply two numbers each of which is a little larger than 100.

To either of the numbers add the supplement of the other; to this sum annex the product of the supplements.

Example. Multiply 131 by 103.

SOLUTION.

$$131 + 3 = 134$$
$$(\text{or } 103 + 31 = 134).$$

Annex 93 (31×3).

The result is 13,493.

The same rule may be applied to numbers a little larger than 1000.

Example. Multiply 1,062 by 1,006.

SOLUTION.

$$1062 + 6 = 1068$$
$$62 \times 6 = \underline{\quad 372}$$
$$1068372$$

NOTE. When the supplements are based on 100 and the product of the supplements is a number of only one digit, a zero must be put in tens' place. For example: $102 \times 103 = 10{,}506$. Similarly, when the supplements are based on 1000 and the product of the supplements is a number of less than three digits, zeros must be put in the proper places.

Written Work

1. 104×120.	2. 127×102.	3. 113×106.
4. 114×105.	5. 114×106.	6. 109×106.
7. 126×107.	8. 109×111.	9. $1{,}007 \times 1{,}003$.

10. $1{,}009 \times 1{,}012$. 11. $1{,}214 \times 1{,}006$. 12. $1{,}206 \times 1{,}012$.

13. $1{,}112 \times 1{,}006$. 14. $1{,}416 \times 1{,}009$. 15. $1{,}374 \times 1{,}005$.

16. $1{,}674 \times 1{,}012$.

27. The **complement** of a number is the difference between the next higher power of 10 and the number. The complement of 92 is 8; the complement of 89 is 11; the complement of 996 is 4.

28. To multiply two numbers both slightly less than 100.

From either number subtract the complement of the other number. To this result annex the product of the complements.

Example. Multiply 96 by 93.

SOLUTION. $96 - 7 = 89$ (or $93 - 4 = 89$).
$4 \times 7 = 28$ (product of the complements).

Annex 28 to 89 and the result is 8928.

NOTE. When the complements are based on 100 and the product of the complements is less than 10, a zero must be put in tens' place. For illustration: $98 \times 97 = 9506$. Similarly, the tens' and hundreds' places must be filled when the complements are based on 1000.

Written Work

Multiply as indicated. Perform orally as much of this work as possible.

1. 98×95. 2. 97×84. 3. 87×91.

4. 95×80. 5. 93×85. 6. 87×97.

7. 88×87. 8. 91×92. 9. 86×94.

The same method may be applied to numbers slightly less than 1000.

Example. Multiply 996 by 987.

SOLUTION. $996 - 13$ (the complement of 987) $= 983$.
$4 \times 13 = 52$ (the product of the complements).
Since the complements are based on 1000 and the product of the complements is a number of only two digits, a zero must be put in hundreds' place. The result is 983,052.

Multiply:

Written Work

1. 994 992	**2.** 996 984	**3.** 983 991	**4.** 987 992
5. 991 981	**6.** 983 982	**7.** 978 983	**8.** 981 988

29. To multiply any two numbers in the *teens*.

To either of the numbers add the units' digit of the other number and annex a zero.

To this result add the product of the units' digits.

Example. Multiply 15 by 17.

SOLUTION. $15 + 7 = 22$; annex a zero, 220

$$\underline{35}$$

$$255$$

Oral Work

Multiply:

1. 17 12	**2.** 13 12	**3.** 16 18	**4.** 17 13	**5.** 14 13	**6.** 19 15
7. 13 16	**8.** 17 19	**9.** 18 12	**10.** 13 19	**11.** 17 16	**12.** 15 17

CHAPTER IV

DIVISION

Oral Work

30. Short Division.

1. Divide by 2 : 16, 28, 76, 248, 368, 926, 1,054.
2. Divide by 3 : 27, 36, 57, 75, 417, 732, 873.
3. Divide by 4 : 32, 48. 64, 72, 96, 196, 384, 748.
4. Divide by 5 : 75, 95, 145, 545, 725, 965, 1,025, 1,370.
5. Divide by 6 : 72, 96, 126, 366, 528, 732, 1,044.
6. Divide by 7 : 56, 91, 147, 203, 476, 924, 1,575.
7. Divide by 8 : 72, 96, 144, 360, 424, 792, 1,240.
8. Divide by 9 : 54, 98, 171, 243, 378, 567, 981.

State the remainder when each of the following numbers is divided by 3, 4, 5, 7, 8, and 9.

9.	10.	11.	12.	13.
473	9,846	2,638	4,020	19,206
692	3,723	7,284	7,302	47,308
876	9,264	3,047	7,960	23,074
479	3,749	2,636	8,403	95,306

14. How many pounds of meat can be bought for $1.26 at 18¢ per pound?

15. How many weeks in 91 days? 175 days? 266 days?

16. The dividend is 176 and the quotient is 8. What is the divisor?

17. The divisor is 9, the quotient is 7, and the remainder is 3. What is the dividend?

18. Six dozen oranges were bought for $1.62. What was the price per dozen?

19. How many yards in 186 feet? 924 feet? 5280 feet?

20. A grocer buys 9 dozen eggs for $2.07. What is the cost per dozen?

21. If you are to discharge a debt of $1.14 in 6 equal weekly payments, how much must you pay each week?

31. Long Division.

Examples. 1. Divide 4,625 by 37. 2. Divide 47,285 by 327.

SOLUTION.

$$\begin{array}{r|l} & 125 \\ 37 & 4625 \\ \hline & 37 \\ \hline & 92 \\ & 74 \\ \hline & 185 \\ & 185 \\ \hline \end{array}$$

SOLUTION.

$$\begin{array}{r|l} & 144\frac{197}{327} \\ 327 & 47285 \\ \hline & 327 \\ \hline & 1458 \\ & 1308 \\ \hline & 1505 \\ & 1308 \\ \hline & 197 \end{array}$$

(The quotient should be written above the dividend in long division.)

32. Checks. Division may be checked by several methods. The method stated below is easily understood and readily applied.

Multiply the quotient by the divisor and add the remainder to this product to obtain the dividend.

Written Work

Perform the following divisions and check each result:

1. $1{,}758 \div 144$.
2. $15{,}762 \div 37$.
3. $73{,}627 \div 425$.
4. $78{,}264 \div 738$.
5. $508{,}573 \div 97$.
6. $66{,}816 \div 928$.
7. $41{,}228 \div 44$.
8. $2{,}235{,}812 \div 284$.
9. Divide the sum of 478, 392, 648, 971, and 1483 by 27.
10. Divide the sum of 3024, 4763, 8297, 9468, and 293 by 38.
11. Complete:

$37 \times 29 =$
$42 \times 29 =$
$36 \times 29 =$
$94 \times 29 =$
$402 \times 29 =$
Total $=$ _____

12. The total circulation of a certain daily newspaper for 23 consecutive days was 524,423 copies. What was the average daily circulation?

13. At $2.18 a yard, how many yards of cloth can be bought for $37.06?

14. The dividend is 14,286, the quotient is 18, and the remainder is 12. What is the divisor?

15. The product of two numbers is 135,468. One of the numbers is 426. What is the other?

16. In a factory 437 men produce 5681 articles in a week. What is the average weekly production for each employee?

17. At the rate of 43 miles an hour, how long will it take a train to run 473 miles?

18. Into how many states as large as Rhode Island (1067 sq. mi.) could Texas (262,398 sq. mi.) be divided?

19. The expenses for 5 months for a family of 5 amounted to $258 for food, $170 for rent, $216 for clothes, $147 for amusements and other expenses. What was the average expense a month for the family and the average per month for each person?

33. Pointing Off the Quotient. When it is necessary to continue a division to tenths or hundredths, place a decimal point to the right of the units' figure of the dividend and the quotient, annex zeros to the dividend, and continue the process.

Example. Divide 2916 by 26.

SOLUTION.

$$26 \overline{)2916.0} \quad 112.1$$

Written Work

The following table gives the area of each of the states in square miles, and also the population of each state.

Review the instructions given on page 9 for ruling a blank form and for entering statistics.

Prepare a form similar to the following:

State and Division	Land Area in Square Miles	Population 1910	Population per Square Mile
New England			
Maine	29,895	742,371	
New Hampshire	9,031	430,572	
Vermont	9,124	355,956	
Massachusetts	8,039	3,366,416	
Rhode Island	1,067	542,610	
Connecticut	4,820	1,114,756	
Middle Atlantic			
New York	47,654	9,113,614	
New Jersey	7,514	2,537,167	
Pennsylvania	44,832	7,665,111	
East North Central			
Ohio	40,740	4,767,121	
Indiana	36,045	2,700,876	
Illinois	56,043	5,638,591	
Michigan	57,480	2,810,173	
Wisconsin	55,256	2,333,860	
West North Central			
Minnesota	80,858	2,075,708	
Iowa	55,586	2,224,771	
Missouri	68,727	3,293,335	
North Dakota	70,183	577,056	
South Dakota	76,868	583,888	
Nebraska	76,808	1,192,214	
Kansas	81,774	1,690,949	
South Atlantic			
Delaware	1,965	202,322	
Maryland	9,941	1,295,346	
D. C.	60	331,069	
Virginia	40,262	2,061,612	
West Virginia	24,022	1,221,119	
North Carolina	48,740	2,206,287	
South Carolina	30,495	1,515,400	
Georgia	58,725	2,609,121	
Florida	54,861	752,619	
East South Central			
Kentucky	40,181	2,289,905	
Tennessee	41,687	2,184,789	
Alabama	51,279	2,138,093	
Mississippi	46,362	1,797,114	

State and Division	Land Area in Square Miles	Population 1910	Population per Square Mile
West South Central			
Arkansas	52,525	1,574,449	
Louisiana	45,409	1,678,339	
Oklahoma	69,414	1,657,155	
Texas	262,398	3,896,542	
Mountain			
Montana	146,201	376,053	
Idaho	83,354	325,594	
Wyoming	97,594	145,965	
Colorado	103,658	799,024	
New Mexico	122,503	327,301	
Arizona	113,810	204,354	
Utah	82,184	373,351	
Nevada	109,821	81,875	
Pacific			
Washington	66,836	1,141,990	
Oregon	95,607	672,765	
California	155,652	2,377,549	

Note. The class may be divided into groups and each group made responsible for a geographical division.

Complete as much of the table as the teacher assigns, showing the population per square mile of each state, of each section, and of the United States.

Carry results to one decimal place.

Exercises for Drill

I. Estimated Time 7 Minutes

1. Add:

a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*	*h.*
27	14	29	73	29	83	72	94
36	33	43	99	48	92	86	55
92	27	38	26	73	17	93	76
23	19	64	10	27	64	14	87
49	86	27	18	65	20	27	31
85	35	85	35	42	78	38	49

2. Multiply:

a.	*b.*	*c.*
4736	9463	7427
3	5	3

3. Subtract:

a.	*b.*	*c.*	*d.*
72049	4706	32964	92807
32608	2985	7193	1968

4. Add across and down:

a.	*b.*
$7+3+9+2=$	$24+7+6=$
$5+4+8+3=$	$13+5+3=$
$4+5+1+9=$	$17+4+7=$
$7+6+5+1=$	$9+15+3=$
$1+9+3+4=$	$18+9+5=$
$8+1+2+5=$	$13+2+1=$
$3+2+7+8=$	$7+19+8=$
$9+7+4+6=$	$15+3+9=$
$+\ +\ +\ =$	$+\ +\ =$

II. Estimated Time 9 Minutes

1. Add across and down:

$$741+437+563+234$$
$$526+318+725+238$$
$$382+746+128+407$$
$$387+209+504+916$$
$$328+460+937+659$$
$$473+295+243+175$$
$$+\quad +\quad +$$

2. Multiply:

a.	*b.*	*c.*	*d.*
7460	9047	739	7036
38	29	86	43

3.

$143 \times 6 =$
$427 \times 5 =$
$793 \times 8 =$
$429 \times 7 =$
$384 \times 3 =$
$827 \times 6 =$
$538 \times 9 =$

4. Divide:

a. $326\overline{)6846}$ *b.* $22\overline{)6996}$

III. Estimated Time 8 Minutes

Add across and down:

7	9	7	3	5	8	1	8
4	2	5	9	2	4	7	3
3	8	3	6	4	5	3	6
6	4	7	7	8	9	8	9
2	3	9	4	7	7	6	7
9	5	8	8	9	6	4	2
8	7	6	5	3	3	5	4
4	6	4	1	1	4	9	8
7	1	9	2	2	1	2	5
6	9	2	8	6	2	7	1

Subtract:

a.	*b.*	*c.*
74036	92407	769082
21809	38795	432767

Multiply:

a.	*b.*	*c.*
3742	9427	3897
37	45	28

Divide:

a. $6\overline{)748632}$ *b.* $8\overline{)938424}$

IV. Estimated Time 8 Minutes

1. Add:

a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*	*h.*
37	24	33	51	26	72	43	18
24	13	15	32	39	46	75	37
36	43	31	44	72	92	76	11
15	25	17	65	43	54	28	26
13	17	65	23	86	28	35	34
11	38	22	25	54	63	54	82

2. Subtract:

a.	*b.*	*c.*	*d.*
4372	7246	9264	3164
2439	3817	5728	1476

3. Add from left to right:

a.	*b.*
$7+3+2+4=$	$6+9+4+8=$
$3+7+5+6=$	$13+17+19+12=$
$4+8+9+1=$	$15+13+12+16=$
$3+7+2+4=$	$5+7+8+6=$
$8+3+7+5=$	$13+27+32+17=$
$9+3+2+8=$	$48+27+3+2=$
$5+7+3+8=$	$7+9+2+3=$

4. Multiply:

a.	*b.*	*c.*	*d.*
3724	4923	6728	3107
2	3	2	3

V. Estimated Time 12 Minutes

1. Divide:

a.	*b.*	*c.*
$5\overline{)742385}$	$8\overline{)943824}$	$9\overline{)736848}$

2. Multiply:

a.	*b.*	*c.*
1437	1943	1048
246	632	453

3. Subtract:

a.	*b.*	*c.*	*d.*
472,387	940,360	472,863	796,048
29,389	397,685	30,940	397,059

4. Add across and down:

$$943 + 790 + 408 + 941 =$$
$$276 + 463 + 312 + 806 =$$
$$841 + 287 + 637 + 420 =$$
$$732 + 934 + 924 + 387 =$$
$$286 + 742 + 836 + 209 = \_\_\_\_$$
$$+ \quad + \quad + \quad =$$

5. Divide:

a. $423\overline{)13536}$

b. $91\overline{)46228}$

Computing Machines of various kinds are generally used by banks and other business houses. Machines on which addition, subtraction, multiplication, and division may be performed are quite common.

CHAPTER V

AVERAGE

34. Simple Average. A boy walked 9 miles on Monday, 5 miles on Tuesday, and 10 miles on Wednesday. If he had divided the trip into three equal distances, how many miles would he have walked each day? In other words, what was the average distance walked each day?

A man owned a five-acre field. One acre yielded 71 bushels of potatoes, another 77 bushels, a third 85 bushels, the fourth 93 bushels, and the fifth 64 bushels. What was the total crop? What was the average crop per acre?

What two processes are usually involved in computing an average?

Written Work

1. The noon temperatures in a certain city for a week were as follows: Monday, 62°; Tuesday, 70°; Wednesday, 74°; Thursday, 68°; Friday, 62°; Saturday, 70°; Sunday, 76°. What was the average noon temperature for the week?

2. A newsdealer sold papers as indicated in the table. Find the average number of each sold.

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Avg.
Sentinel . . .	39	47	63	82	74	67	
Argus	13	19	27	31	29	26	
Tribune . . .	39	42	48	51	39	41	
Recorder . . .	28	43	56	29	37	29	

3. The following table gives the prices received by farmers in certain states for the butter sold during the months of a recent year. Find the average price received in each state during the year.

BUTTER PRICES IN CERTAIN STATES

	CENTS PER POUND												AVERAGE PRICE BY STATES
	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	
Massachusetts	37	38	36	35	33	34	31	33	34	34	32	35	
Rhode Island	35	39	39	34	34	36	32	32	35	34	34	34	
Connecticut	36	39	38	35	34	35	33	34	34	36	34	36	
New York	34	35	32	31	31	30	27	29	29	30	32	35	
New Jersey	37	40	35	34	34	34	32	32	33	32	34	36	
Pennsylvania	33	35	33	31	31	29	26	27	28	30	32	34	
Maryland	28	29	29	28	25	25	25	25	26	28	29	28	
Virginia	25	26	26	26	25	23	21	22	22	24	26	26	
West Virginia	26	26	26	26	26	22	21	21	22	24	25	27	
Georgia	25	25	28	24	24	24	24	24	24	25	25	26	
Ohio	27	28	27	25	25	24	22	23	24	25	27	29	
Indiana	25	26	25	24	24	22	21	22	22	24	25	27	
Illinois	27	28	26	25	25	24	24	23	24	26	26	28	
Michigan	30	31	28	27	27	25	23	23	24	25	27	29	
Wisconsin	33	34	28	28	29	26	25	25	26	27	28	31	
Minnesota	31	32	29	27	27	27	24	24	25	26	28	30	
Iowa	29	30	27	26	26	25	24	24	24	25	27	29	
Missouri	23	23	23	23	23	22	21	21	21	22	23	24	
Nebraska	26	26	24	24	23	22	21	21	22	23	25	27	
Kansas	26	26	25	24	24	22	21	22	22	24	25	26	
Kentucky	21	22	21	21	21	21	19	19	18	20	20	23	
Tennessee	21	22	21	20	20	19	18	18	18	19	20	22	
Louisiana	28	30	28	27	28	27	26	27	27	27	28	30	
Texas	22	25	23	22	21	21	21	20	21	23	23	24	
Oklahoma	27	25	23	22	22	22	20	20	19	23	24	25	
Montana	36	37	35	33	31	31	30	29	31	31	32	35	
Colorado	33	33	30	30	28	28	26	26	28	28	32	31	
Utah	33	31	30	29	31	30	29	27	28	30	32	32	
Idaho	35	33	32	32	31	28	28	27	29	31	32	34	
Washington	36	37	32	32	30	28	28	30	30	32	33	35	
Oregon	34	35	33	32	31	26	28	28	30	30	35	36	
California	34	36	34	32	30	29	29	31	31	33	34	36	

4. The following table gives the acreage planted to certain crops in the United States, and the crops raised. Compute the average yield per acre. Carry your results to two decimal places. Show the statistics on a ruled form.

ACREAGE AND PRODUCTION OF CROPS

CROP	1911			1912		
	1000 Acres	1000 Bushels	Bushels per Acre	1000 Acres	1000 Bushels	Bushels per Acre
Corn . .	105,825	2,531,488		107,083	3,124,746	
Wheat . .	49,543	621,338		45,814	730,267	
Oats . .	37,763	922,298		37,917	1,418,337	
Rye . . .	2,127	33,119		2,117	35,664	
Potatoes .	3,619	292,737		3,711	420,647	

5. The following table shows the number of cases of eggs shipped to seven leading markets in the United States.

RECEIPTS OF EGGS AT SEVEN LEADING MARKETS IN THE UNITED STATES
1906–1912

YEAR	BOSTON	CHICAGO	CINCINNATI	MILWAUKEE	NEW YORK	ST. LOUIS	SAN FRANCISCO	TOTAL
1906	1,709,531	3,583,878	484,208	187,561	3,981,013	1,023,125	137,074	
1907	1,594,576	4,780,356	588,636	176,826	4,262,153	1,288,977	379,429	
1908	1,436,786	4,569,014	441,072	207,558	3,703,990	1,439,868	347,436	
1909	1,417,397	4,557,906	519,652	160,418	3,903,867	1,395,987	340,185	
1910	1,431,686	4,844,045	511,519	169,448	4,380,777	1,375,638	469,698	
Av. 1906–1910								
1911	1,441,748	4,707,335	605,131	175,270	5,021,757	1,736,915	587,115	
1912	1,580,106	4,556,643	668,942	136,621	4,723,558	1,391,611	638,920	
Av. 1911–1912								

Find (1) the total receipts of eggs each year, (2) the average receipts at each city for the periods indicated.

6. A subscription was taken to secure funds to purchase a gift.

3 men gave $1.00 each
1 man gave .75
2 men gave 1.50 each
1 man gave .60
4 men gave .90 each

Find the average amount given.

7. The following table shows the number of men employed and the total weekly wages in each of the four departments of a factory. Find the average wage in each department and the average wage for the four departments.

Department	Number of Men Employed	Total Wage	Average Wage
A	47	$ 960	
B	18	290	
C	62	1054	
D	26	318	

Oral Review

1. What is the difference in the meaning of the following statements?

a. Each of the 300 employees in our factory earns $2.50 per day.

b. The average daily wages of employees in our factory is $2.50 per day.

2. What would you have to know and how would you proceed to find:

a. The average weight of twenty boxes?

b. The average value of a herd of cattle?

c. The average number of miles a train traveled per hour, going from Chicago to St. Louis?

d. The average age of the students in your class?

e. The average daily sales of a clerk?

3. What could you find if you were told:

a. The average value of farm land per acre in your state, and the number of acres of farm land?

b. The average daily sales of a clerk during the twenty-five week days of June?

c. The average monthly grocery bill of your family?

4. If you were told :

a. The daily circulation of the *Evening Herald* for the twenty-six week days of July, and the average daily circulation of the *Evening Transcript* for the same month, how could you compare their circulation ?

b. The average daily wages of A, who worked 265 days last year and the average daily wages of B, who worked 303 days, how could you find the yearly wages of each, and which one earned the larger amount ?

c. The difference between the average daily outputs of two shoe factories, how could you find the difference in their production for a year of 300 working days ?

5. What would you have to know and what would you do to find the average daily speed of an ocean liner on a given voyage ?

6. Several boys worked for a farmer picking strawberries. One earned $5.00 more than the average of the other boys' earnings. What else must you know and what would you do to find the total earnings of all the boys ? What else would you have to know and how would you find the average wage cost of picking a quart of strawberries ?

Written Work

1. A merchant kept a record of the deliveries of goods made to his customers for a week. The record follows:

Day	Deliveries
Monday	213
Tuesday	187
Wednesday	208
Thursday	221
Friday	168
Saturday	251

a. What was the average number of deliveries per day ?

b. The expense of running the delivery wagons for a week, including care of horses, interest on the money invested in horses and wagons, repairs and wages, was $64.75. What was the average cost per delivery ?

2. Five clerks in a store sold the following amounts of goods during a month:

A.	$1246.50
B.	1076.85
C.	944.90
D.	1388.20
E.	1109.75

a. What was the average amount of sales per clerk?

b. Which clerks sold more than the average? Which clerks sold less than the average?

3. The distance from Chicago to Aurora, Illinois, via the C. B. & Q. Railroad, is 37.4 miles. Train No. 55 makes no stops between these stations. It leaves Chicago at 6 : 10 P.M. and arrives at Aurora at 7 : 10 P.M. How many miles an hour does this train travel? What fractional part of a mile does it run in one minute?

4. The single fare between these two stations is 74 cents. Ten-trip tickets may be purchased for $6.25. What is the saving in fare per trip?

5. A 25-trip ticket may be purchased for $13.00. What is the average cost per trip?

6. The following table shows the number and value of pianos and organs manufactured in the United States in 1904 and 1909.

	1909	1904
PIANOS		
Number	261,197	374,154
Value	$41,476,479	$59,501,225
ORGANS		
Number	113,065	64,111
Value	$4,162,053	$2,595,429

What was the average value of a piano manufactured in each of the years?

What was the average value of an organ manufactured in each of the years?

7. In 1900 the population of the United States was, in round numbers, 77 million. The combined daily circulation of all daily newspapers was about 15 million copies; an average of 1 copy of a daily paper to every ------- persons. In 1910 the population had increased to 93 million, and the circulation of daily newpapers to 24 million, or an average of one copy for every -------- persons.

8. During the year 1913 the United States Congress appropriated $1,098,678,788 for the expenses of the government. The last preceding census showed a population of 93,402,151. What was the average governmental expenditure per person on this basis?

CHAPTER VI

FACTORS AND MULTIPLES

35. Terms. An **integer** is a number of whole units.

The **factors** of a number are the integers which, multiplied together, produce the given number. Thus, the factors of 15 are 3 and 5; the factors of 18 are 3 and 6, or 2 and 9.

A **factor** of a number is a divisor of that number.

A number which is not exactly divisible by any other number (except 1) is called a **prime number**. Thus, 1, 3, 5, 7, 11, and 13 are prime numbers.

Numbers are said to be **prime to each other** when they have no common factor except 1. Thus, 10 and 27 are **prime to each other**, although neither is a prime number.

36. Test of Divisibility of Numbers. A number is divisible by:

a. Two, if it ends with 0, 2, 4, 6, or 8.

b. Three, if the sum of its digits is divisible by 3.

c. Four, if the number expressed by its last two digits is divisible by 4.

d. Five, if it ends in 0 or 5.

e. Six, if it is divisible by both 2 and 3.

f. Eight, if the number expressed by its last three digits is divisible by 8.

g. Nine, if the sum of its digits is divisible by 9.

h. Ten, if its right-hand digit is zero.

i. Eleven, if the difference between the sums of the numbers represented by the odd and even orders of digits is divisible by 11. Thus, 16,280 is divisible by 11, since $(8+6)-(0+2+1)$ is divisible by 11.

(There is no simple method of testing divisibility by 7.)

37. Factoring is the process of separating a number into its factors.

Oral Work

1. Learn the prime numbers from 1 to 100 so that you can recognize them at sight.

2. Apply the tests of divisibility to the following.

Find the *prime* factors of :

28	160	728	478	76	720
42	320	640	96	84	37
72	48	386	84	90	145
36	360	31	92	360	390
98	280	100	81	760	625

3. What numbers between 161 and 200 are divisible by 9?

4. What numbers between 746 and 800 are divisible by 6?

5. Name the factors of 36 which are not prime to each other.

38. Cancellation is the process of shortening certain computations involving division by removing or canceling equal factors from both dividend and divisor.

Example. Divide the product of 4, 9, 8, 36, 24, and 7 by the product of 18, 2, 8, 3, 14, and 4.

SOLUTION.
$$\frac{\overset{2}{\cancel{4}} \times \overset{1}{\cancel{9}} \times \overset{1}{\cancel{8}} \times \overset{18}{\cancel{36}} \times \overset{12}{\cancel{24}} \times 7}{\underset{2}{\cancel{18}} \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{8}} \times \overset{1}{\cancel{3}} \times \underset{2}{\cancel{14}} \times \overset{1}{\cancel{4}}} = 36.$$

In all computations involving only multiplication and division, cancellation should be used when possible. Indicate the multiplication and division as in the illustration above, then cancel the common factors.

Written Work

(Use cancellation when possible.)

1. $\dfrac{24 \times 36 \times 15}{16 \times 5 \times 9}$.

2. $\dfrac{4 \times 37 \times 16 \times 5 \times 60}{48 \times 32 \times 74}$.

3. $\dfrac{48 \times 32 \times 100 \times 360}{16 \times 50 \times 72}$.

4. $\dfrac{27 \times 64 \times 96 \times 38}{19 \times 16 \times 9 \times 2}$.

5. $\dfrac{130 \times 14 \times 18 \times 121 \times 15}{7 \times 27 \times 13 \times 11}$.

6. $\dfrac{144 \times 32 \times 63 \times 7}{16 \times 9 \times 28}$.

7. $\dfrac{1728 \times 360 \times 100 \times 32 \times 3}{18 \times 144 \times 64 \times 75}$.

8. $\dfrac{21 \times 72 \times 160 \times 340 \times 27}{180 \times 36 \times 35}$.

39. Greatest Common Divisor. An integer that is a factor of two or more numbers is called a **common divisor**, or a **common factor** of those numbers.

The **greatest common divisor** of two or more numbers is the greatest factor common to the numbers. "Greatest common divisor" is usually expressed as g. c. d.

Example. Find the greatest common divisor of 12, 20, and 36.

SOLUTION.
$$12 = 2 \times 2 \times 3$$
$$20 = 2 \times 2 \times 5$$
$$36 = 2 \times 2 \times 3 \times 3.$$

The factor 2 occurs twice in all the numbers and none of the other factors occurs in all the numbers, hence, 4 is the g. c. d. of 12, 20, and 36.

To find the g. c. d. of two or more numbers, *separate the numbers into prime factors and find the product of the prime factors common to the numbers.*

Written Work

Find the g. c. d. of:

1. 12, 18, 24.
2. 24, 60, 72.
3. 15, 20, 30.
4. 60, 90, 100.
5. 84, 32, 60.
6. 60, 96, 120.
7. 18, 32, 48.
8. 27, 36, 45.
9. 360, 120, 40.
10. 121, 88, 242.
11. Find the g. c. d. of 8 ft. and 12 ft.
12. Find the g. c. d. of \$48 and \$60.

40. Least Common Multiple. A **multiple** of a number is an integral number of times that number. Thus, 28 is a multiple of 7. 60 is a multiple of 12.

A **common multiple** of two or more numbers is a number that is a multiple of each of them. It is therefore divisible by each of them.

The **least common multiple** of two or more numbers is the least number that is a multiple of each of them.

Thus, 60 is the least common multiple of 12, 15, and 30.

Example. Find the least common multiple (l. c. m.) of 18, 20, and 24.

SOLUTION.

$$18 = 2 \times 3 \times 3$$
$$20 = 2 \times 2 \times 5$$
$$24 = 2 \times 2 \times 2 \times 3.$$
$$\text{l. c. m.} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360.$$

To find the least common multiple of two or more numbers, *separate each number into its prime factors. Find the product of these factors, using each factor the greatest number of times it occurs in any one of the given numbers.*

Written Work

Find the l. c. m. of the following:

1. 12, 15.
2. 8, 12.
3. 6, 15.
4. 7, 8, 12.
5. 8, 9, 12, 15.
6. 24, 36, 60.
7. 8, 12, 16.
8. 36, 24, 75.
9. 360, 345.
10. 75, 130, 190.
11. 425, 345, 336.
12. 360, 240, 420.

Find the g. c. d. and the l. c. m. of the following:

13. 60, 80, 95.
14. 36, 75, 48.
15. 480, 360, 120.

CHAPTER VII

COMMON FRACTIONS

41. Terms. A **fraction** is one or more equal parts of a unit.

A **common fraction** is usually expressed by writing one figure above and one below a short line; thus, $\frac{2}{3}$.

The **numerator** of a fraction is the number which shows how many of the equal parts of the unit are taken. It is written above the line.

The **denominator** of a fraction is the number which shows into how many equal parts the unit is divided. It is written below the line.

The numerator and the denominator are called the **terms** of the fraction. Thus, 3 and 4 are the terms of the fraction $\frac{3}{4}$.

A common fraction may be either proper or improper.

A **proper fraction** is one whose numerator is less than its denominator, as $\frac{3}{4}$.

An **improper fraction** is one whose numerator is equal to or greater than its denominator, as $\frac{4}{4}$ or $\frac{5}{4}$.

A **mixed number** consists of a whole number and a fraction, as $5\frac{2}{3}$.

Oral Work

Which is greater, $\frac{1}{4}$ or $\frac{1}{7}$? $\frac{1}{9}$ or $\frac{1}{10}$? $\frac{1}{20}$ or $\frac{1}{12}$?

How is the value of a fraction affected by increasing the numerator, the denominator remaining the same?

Reduction of Fractions

42. Reducing Fractions to Lower Terms. When the numerator and the denominator contain one or more common factors, the fraction may be reduced to a fraction of equivalent value expressed in lower terms.

Thus, $\frac{10}{12}$ may be reduced to the equivalent fraction $\frac{5}{6}$, by dividing both terms by 2. Similarly, $\frac{18}{24} = \frac{3}{4}$; $\frac{7}{21} = \frac{1}{3}$.

State a rule for reducing fractions to lower terms.

43. Reducing Fractions to Higher Terms. Fractions may be raised to equivalent fractions in higher terms, *by multiplying both the numerator and the denominator by the same number.*

Example. Express $\frac{3}{5}$ as an equivalent fraction whose denominator is 60.

SOLUTION. The given denominator, 5, must be multiplied by 12 to obtain the desired denominator, 60. Therefore, multiply both terms of the fraction by 12, and obtain the equivalent fraction $\frac{36}{60}$.

Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.

44. Reducing Improper Fractions to Mixed Numbers. An improper fraction may be reduced to a mixed number. The following example shows the method:

Example. Reduce $\frac{19}{4}$ to a mixed number.

SOLUTION. $19 \div 4 = 4\frac{3}{4}$.

State a rule for changing an improper fraction to a mixed number.

45. Reducing Mixed Numbers to Improper Fractions. A mixed number may be reduced to an improper fraction. The following example shows the method.

Example. Reduce $13\frac{3}{7}$ to an improper fraction.

SOLUTION. $13 \times 7 = 91$. $91 + 3 = 94$, the numerator of the improper fraction.

The denominator of the fraction in the mixed number is retained as the denominator of the improper fraction.

$$13\tfrac{3}{7} = \tfrac{94}{7}.$$

Oral Work

1. Reduce to lowest terms: $\frac{4}{12}$, $\frac{3}{18}$, $\frac{5}{10}$, $\frac{6}{24}$, $\frac{18}{30}$, $\frac{24}{28}$, $\frac{15}{30}$, $\frac{72}{84}$.

2. Express each of the following fractions with the denominator 16: $\frac{5}{8}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{4}{8}$, $\frac{9}{4}$, $\frac{7}{8}$, $\frac{5}{2}$, $\frac{5}{1}$, 3.

3. Express with the denominator 36: $\frac{4}{9}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{7}{12}$, $\frac{4}{12}$, $\frac{1}{3}$, $\frac{8}{9}$, $\frac{7}{3}$, $\frac{8}{4}$, $\frac{1}{9}$, $\frac{7}{2}$.

4. Reduce to twenty-fourths: $\frac{5}{6}$, $\frac{7}{12}$, $\frac{2}{3}$, $\frac{3}{2}$, $\frac{7}{6}$, $\frac{1}{12}$, 2.

5. Reduce to lowest terms: $\frac{8}{12}$, $\frac{9}{15}$, $\frac{4}{16}$, $\frac{8}{10}$, $\frac{9}{12}$, $\frac{14}{16}$.

6. Reduce to fiftieths: $\frac{2}{5}$, $\frac{9}{10}$, $\frac{7}{25}$, $\frac{19}{10}$, $\frac{5}{2}$, $\frac{7}{10}$.

7. Reduce to improper fractions: $3\frac{1}{5}$, $5\frac{1}{3}$, $7\frac{1}{4}$, $8\frac{2}{3}$, $4\frac{3}{5}$, $7\frac{2}{7}$, $9\frac{4}{5}$, $8\frac{1}{4}$.

8. Reduce the following fractions to equivalent fractions in lowest terms: $\frac{9}{24}$, $\frac{16}{68}$, $\frac{20}{85}$, $\frac{18}{144}$, $\frac{28}{210}$, $\frac{27}{63}$, $\frac{30}{150}$, $\frac{21}{15}$, $\frac{72}{180}$, $\frac{24}{36}$, $\frac{8}{12}$, $\frac{18}{21}$.

9. Change the following fractions as indicated:

$\frac{3}{4}$ to an equivalent fraction whose denominator is 20.
$\frac{7}{8}$ to an equivalent fraction whose denominator is 64.
$\frac{4}{9}$ to an equivalent fraction whose denominator is 63.
$\frac{2}{18}$ to an equivalent fraction whose denominator is 72.
$\frac{6}{11}$ to an equivalent fraction whose denominator is 121.
$\frac{5}{14}$ to an equivalent fraction whose denominator is 84.

10. Change the following improper fractions to mixed numbers: $\frac{8}{3}$, $\frac{17}{6}$, $\frac{39}{7}$, $\frac{25}{9}$, $\frac{31}{2}$, $\frac{84}{11}$, $\frac{488}{15}$, $\frac{300}{29}$, $\frac{341}{17}$, $\frac{83}{13}$, $\frac{216}{13}$, $\frac{235}{12}$.

11. Change the following mixed numbers to equivalent improper fractions: $3\frac{2}{3}$, $6\frac{4}{7}$, $8\frac{1}{8}$, $4\frac{3}{5}$, $8\frac{16}{25}$, $10\frac{17}{35}$, $14\frac{8}{13}$, $12\frac{7}{12}$, $18\frac{3}{20}$, $9\frac{5}{24}$.

12. When we reduce $\frac{8}{12}$ to $\frac{2}{3}$, do we increase or decrease the size of the fractional unit? Do we increase or decrease the number of fractional units?

13. Which fraction is the greatest, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{13}{15}$?

Addition and Subtraction of Fractions

46. Finding the Least Common Denominator. Only like quantities can be added or subtracted. It is not possible to add or subtract 4 lb. and 2 oz. or $\frac{1}{2}$ and $\frac{1}{4}$ until they are reduced to the same denomination.

The common denominator should be the smallest number which will exactly contain all of the denominators. It may usually be found by inspection.

Any number which contains all of the factors of another number

will also contain that number. Thus, 24 contains all of the factors of 12, and it therefore contains 12.

The common denominator must, therefore, contain all of the factors of the denominators.

Hence, to find the least common denominator of two or more fractions, *find the prime factors of the denominators; multiply the different prime factors, using each factor the greatest number of times it is contained in any one denominator.*

Example. What is the least common denominator of fractions with the denominators 8, 9, and 12?

SOLUTION.

$8 = 2 \times 2 \times 2.$
$9 = 3 \times 3.$
$12 = 2 \times 2 \times 3.$
l. c. m. of denominators $= 2 \times 2 \times 2 \times 3 \times 3.$

47. Reducing Fractions to a Common Denominator. After the least common denominator has been found, the fractions may be reduced to this denominator.

Example. Reduce $\frac{3}{5}$, $\frac{2}{3}$, and $\frac{5}{6}$ to a common denominator.

SOLUTION. The common denominator is 30.

$\frac{3}{5} = \frac{18}{30}.$
$\frac{2}{3} = \frac{20}{30}.$
$\frac{5}{6} = \frac{25}{30}.$

State the process of reducing fractions to a common denominator.

Written Work

Reduce the fractions in each example to fractions having the least common denominator:

1. $\frac{1}{2}$ and $\frac{1}{3}$.
2. $\frac{1}{3}$ and $\frac{1}{4}$.
3. $\frac{1}{4}$ and $\frac{1}{8}$.
4. $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{1}{12}$.
5. $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{20}$.
6. $\frac{2}{3}$, $\frac{4}{9}$, $\frac{5}{12}$.
7. $\frac{3}{7}$, $\frac{2}{11}$, $\frac{5}{42}$.
8. $\frac{7}{12}$, $\frac{9}{32}$, $\frac{13}{56}$, $\frac{4}{9}$, $\frac{5}{14}$.
9. $\frac{3}{20}$, $\frac{17}{36}$, $\frac{8}{49}$, $\frac{12}{35}$.
10. $\frac{11}{18}$, $\frac{9}{28}$, $\frac{1}{42}$, $\frac{11}{15}$.
11. $\frac{17}{96}$, $\frac{23}{42}$, $\frac{25}{63}$, $\frac{19}{27}$.

48. Rule for Addition of Fractions.

Reduce the fractions to a common denominator. Add the numerators to form the numerator of the sum. The common denominator is the denominator of the sum. Reduce the result to the simplest form.

Proper fractions should be reduced to their lowest terms. Improper fractions should be changed to mixed numbers.

Examples. 1. Add $\frac{2}{3}$ and $\frac{3}{4}$.

SOLUTION. The least common multiple of the denominators is 12.

$$\tfrac{2}{3} = \tfrac{8}{12}.$$
$$\tfrac{3}{4} = \tfrac{9}{12}.$$
$$\tfrac{8}{12} + \tfrac{9}{12} = \tfrac{17}{12} = 1\tfrac{5}{12}.$$

2. Add $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{10}$.

SOLUTION. The least common multiple of the denominators is 60.

$$\tfrac{3}{4} = \tfrac{45}{60}.$$
$$\tfrac{5}{6} = \tfrac{50}{60}.$$
$$\tfrac{7}{10} = \tfrac{42}{60}.$$
$$\tfrac{45}{60} + \tfrac{50}{60} + \tfrac{42}{60} = \tfrac{137}{60} = 2\tfrac{17}{60}.$$

49. Rule for Subtraction of Fractions.

The process is the same as for Addition of Fractions except that the numerators are subtracted instead of added after the fractions have been reduced to a common denominator.

Oral Work

State the sum, then the difference of the following:

1. $\frac{1}{2}, \frac{1}{3}$.	**2.** $\frac{1}{3}, \frac{1}{4}$.	**3.** $\frac{1}{5}, \frac{1}{7}$.	**4.** $\frac{1}{4}, \frac{1}{6}$.	**5.** $\frac{1}{3}, \frac{1}{5}$.
6. $\frac{1}{2}, \frac{1}{8}$.	**7.** $\frac{1}{7}, \frac{1}{9}$.	**8.** $\frac{1}{9}, \frac{1}{10}$.	**9.** $\frac{2}{3}, \frac{2}{7}$.	**10.** $\frac{5}{6}, \frac{3}{4}$.
11. $\frac{7}{9}, \frac{2}{3}$.	**12.** $\frac{5}{6}, \frac{4}{5}$.	**13.** $\frac{4}{7}, \frac{1}{3}$.	**14.** $\frac{3}{4}, \frac{3}{5}$.	**15.** $\frac{3}{5}, \frac{2}{7}$.

NOTE. The Short Methods for the Addition and Subtraction of Fractions, explained on pages 69–71, may be studied at this time.

50. Rule for Addition of Mixed Numbers.

Add the fractions.

If the sum is an improper fraction, reduce it to a mixed number.

Add the sum of the fractions to the sum of the integers.

Example. Add $3\frac{3}{4}$ and $4\frac{7}{8}$.

SOLUTION.
$$\tfrac{3}{4} + \tfrac{7}{8} = \tfrac{13}{8} = 1\tfrac{5}{8}.$$
$$3 + 4 = 7.$$
$$7 + 1\tfrac{5}{8} = 8\tfrac{5}{8}.$$

Written Work

Find the sums:

1. $4\frac{1}{2} + 3\frac{1}{3}$.
2. $2\frac{1}{5} + 5\frac{1}{7}$.
3. $7\frac{1}{8} + 5\frac{2}{3}$.
4. $17\frac{1}{5} + 18\frac{2}{7}$.
5. $275\frac{2}{3} + 78\frac{1}{5}$.
6. $3\frac{7}{8} + 5\frac{1}{4}$.
7. $9\frac{1}{10} + 17\frac{2}{5}$.
8. $3\frac{1}{3} + 4\frac{7}{8} + 3\frac{5}{12}$.
9. $23\frac{1}{3} + 16\frac{5}{12} + 7\frac{3}{4}$.
10. $18\frac{5}{6} + 42\frac{1}{10} + 37\frac{3}{5}$.

51. Rule for Subtraction of Mixed Numbers.

If the fraction of the minuend is larger than that of the subtrahend, the above rule for addition can be easily changed to apply to subtraction.

Example. Subtract $4\frac{2}{5}$ from $7\frac{5}{6}$.

SOLUTION. $\frac{5}{6} - \frac{2}{5} = \frac{13}{30}$.
$7 - 4 = 3$.
Result $3\frac{13}{30}$.

If the fraction of the minuend is smaller than that of the subtrahend, we may proceed as follows:

Example. Subtract $4\frac{2}{3}$ from $9\frac{1}{5}$.

SOLUTION. $\frac{2}{3}$ cannot be subtracted from $\frac{1}{5}$, but $9\frac{1}{5}$ may be regarded as $8\frac{6}{5}$. $\frac{6}{5} - \frac{2}{3} = \frac{8}{15}$. Therefore, $9\frac{1}{5} - 4\frac{2}{3} = 4\frac{8}{15}$.

Written Work

Add as indicated:

1. $\frac{3}{7} + \frac{2}{5}$.
2. $\frac{6}{13} + \frac{8}{9}$.
3. $\frac{7}{9} + \frac{4}{15}$.
4. $\frac{2}{15} + \frac{7}{24}$.
5. $\frac{3}{5} + \frac{9}{25} + \frac{9}{16}$.
6. $\frac{15}{32} + \frac{7}{24} + \frac{11}{36}$.
7. $3\frac{5}{8} + \frac{1}{7}$.
8. $12\frac{9}{16} + \frac{7}{18}$.
9. $13\frac{5}{7} + 22\frac{9}{15}$.

Subtract as indicated:

10. $\frac{4}{7} - \frac{3}{11}$.
11. $\frac{15}{32} - \frac{7}{20}$.
12. $2\frac{9}{14} - \frac{3}{16}$.
13. $4\frac{7}{8} - \frac{6}{7}$.
14. $3\frac{2}{3} - 2\frac{1}{2}$.
15. $4\frac{7}{9} - 2\frac{1}{6}$.
16. $5\frac{9}{25} - 2\frac{8}{9}$.

Add upward and across:

17.

$$\begin{array}{ccc} \frac{1}{4} & + \frac{1}{3} & + \frac{1}{2} \\ \frac{2}{3} & + \frac{3}{4} & + \frac{5}{12} \\ \frac{5}{6} & + \frac{7}{8} & + \frac{7}{6} \\ \hline ? & + ? & + ? \end{array}$$

18.

$$\begin{array}{ccc} \frac{5}{8} & + \frac{3}{16} & + \frac{2}{3} \\ \frac{7}{24} & + \frac{5}{6} & + \frac{1}{24} \\ \frac{1}{8} & + \frac{5}{6} & + \frac{7}{4} \\ \hline ? & + ? & + ? \end{array}$$

19. Add upward; subtract across:

$$\begin{array}{r} 14\frac{2}{3} - 4\frac{1}{5} \\ 18\frac{5}{6} - 7\frac{2}{3} \\ 5\frac{1}{12} - 3\frac{1}{4} \\ 15\frac{3}{4} - 7\frac{5}{6} \\ \hline ? \quad - ? \end{array}$$

Multiplication and Division of Fractions

52. Rule for Multiplication of Fractions.

Multiply the numerators to form the numerator of the product. Multiply the denominators to form the denominator of the product. Reduce the product to its simplest form.

Thus, $\frac{3}{5} \times \frac{4}{7} = \frac{12}{35}$.

Note. The word "of" placed between fractions indicates multiplication.

Mixed numbers should be changed to improper fractions before multiplying.

Example. $8 \times 6\frac{3}{4} \times \frac{2}{3} \times 3\frac{4}{5} = ?$

Solution. Reducing mixed numbers to improper fractions: $\frac{8}{1} \times \frac{27}{4} \times \frac{2}{3} \times \frac{19}{5} = ?$

$$\frac{\overset{2}{\cancel{8}}}{1} \times \frac{27}{\underset{1}{\cancel{4}}} \times \frac{2}{\underset{1}{\cancel{3}}} \times \frac{19}{5}.$$

Multiplying the resulting numerators: $2 \times 9 \times 2 \times 19 = 684$, the numerator of the product.

Multiplying the resulting denominators: $1 \times 1 \times 1 \times 5 = 5$, the denominator of the product.

Changing to a mixed number: $\frac{684}{5} = 136\frac{4}{5}$, the product.

Written Work

Perform the following multiplications:

1. $\frac{2}{3} \times \frac{4}{5}$.	**2.** $\frac{3}{8} \times \frac{5}{7}$.	**3.** $\frac{6}{5} \times \frac{8}{7}$.
4. $\frac{1}{2} \times \frac{5}{12}$.	**5.** $\frac{3}{4} \times \frac{7}{8}$.	**6.** $3\frac{1}{5} \times 4\frac{2}{3}$.
7. $\frac{8}{9} \times \frac{3}{4}$.	**8.** $\frac{2}{3}$ of $\frac{3}{4} \times \frac{1}{2}$.	**9.** $\frac{3}{7} \times \frac{3}{5}$ of $\frac{4}{3}$.
10. $\frac{8}{12} \times \frac{5}{7} \times \frac{4}{9} \times \frac{3}{10}$.	**11.** $5\frac{2}{3} \times 3\frac{1}{4} \times 4\frac{2}{5}$.	**12.** $\frac{3}{8} \times \frac{4}{1} \times \frac{7}{2} \times \frac{3}{15}$.

53. Rule for Division of Fractions.

Invert the divisor, and multiply the fractions, using cancellation when possible.

Examples. 1. $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = 1\frac{4}{15}$.

2. $\frac{8}{9} \div \frac{4}{5} = \frac{8}{9} \times \frac{5}{4} = \frac{10}{9} = 1\frac{1}{9}$.

3. $8\frac{2}{3} \div \frac{4}{9} = ?$

SOLUTION. $8\frac{2}{3} \div \frac{4}{9} = \frac{\overset{13}{\cancel{26}}}{\underset{3}{\cancel{3}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{4}}} = \frac{39}{2} = 19\frac{1}{2}$.

Written Work

1. $\frac{3}{5} \div \frac{2}{3}$.
2. $\frac{4}{7} \times \frac{9}{8}$.
3. $\frac{2}{3}$ of $\frac{3}{4} \times \frac{1}{6}$.
4. $5\frac{1}{2} \div 4\frac{2}{3}$.
5. $7\frac{1}{8} \div 3\frac{1}{4}$.
6. $\frac{3}{8} + \frac{2}{3}$.
7. $\frac{4}{7} - \frac{2}{5}$.
8. $3\frac{1}{5} + 5\frac{2}{3}$.
9. $\frac{3}{4}$ of $\frac{3}{7}$.
10. $9\frac{1}{2} \times 12\frac{2}{3} \div 3\frac{1}{4}$.
11. $\frac{7}{8}$ of $\frac{2}{11} \div 1\frac{4}{22}$.
12. $\frac{3}{5} + \frac{2}{9}$.
13. $4\frac{2}{5} - 3\frac{1}{3} + 5\frac{3}{4}$.
14. $17\frac{2}{3} \times 5\frac{1}{2} \div 4\frac{5}{12}$.
15. $\frac{3}{8}$ of $\frac{2}{3} \div \frac{2}{5}$ of $\frac{4}{9}$.

Review Exercises

1. Multiply each of the fractions in the column at the left by each of the fractions at the top. Enter the products in the spaces formed by the horizontal and vertical lines. (For example, the space marked "X" is to contain the product of $\frac{5}{8}$ and $\frac{7}{8}$.)

MULTIPLY	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{7}{8}$
5				
6				
$3\frac{1}{3}$				
$\frac{5}{8}$				×
$\frac{7}{8}$				

2. In the following form divide each of the fractions in the column at the left by the fractions in the upper row. Rule a blank similar to the form given and record the quotients in the proper spaces.

	$\frac{3}{5}$	$\frac{7}{8}$
$\frac{5}{6}$		
$\frac{8}{3}$		
$\frac{3}{4}$		
$\frac{2}{5}$		
$\frac{4}{9}$		
$\frac{5}{4}$		
$\frac{3}{8}$		
$\frac{5}{12}$		
$\frac{8}{9}$		
$\frac{5}{6}$		

Miscellaneous Problems

1. If $\frac{1}{3}$ of a number is 5, what is the number?

2. If $\frac{2}{3}$ of a number is 12, what is the number?

3. If $\frac{4}{7}$ of a number is 20, what is the number?

4. If $\frac{3}{5}$ of $\frac{2}{3}$ of a number is 15, what is the number?

5. After spending $\frac{1}{4}$ of his monthly salary for board and room, and $\frac{1}{10}$ of it for clothes, a man has $65 left. What is his salary?

6. If 5 yards of cloth cost $47\frac{1}{2}$¢, what will 1 yard cost at the same rate?

7. A train runs $\frac{2}{3}$ of a mile in $\frac{5}{6}$ of a minute. What is the rate per hour?

8. Which will give the larger result, multiplying or dividing an integer by a proper fraction? Multiplying or dividing by an improper fraction?

9. If you are told the amount of a man's wages and the fractional part of his wages which he spent, how can you find the fractional part which he saved?

10. State two ways of finding the amount saved, and state which you think is the easier.

11. If you were told what $\frac{3}{4}$ of $\frac{2}{3}$ of a number is, how could you find the number?

12. A man owned $\frac{3}{4}$ of a farm and sold $\frac{1}{5}$ of what he owned. If he received $3000 for the land sold, what was the value of the entire farm at the same rate?

13. A mechanic works 8 hours a day. Two and one half hours are spent at a bench and the remainder at a machine. What fractional part of the day is spent at a machine?

14. In a certain family $\frac{1}{4}$ of the yearly income is spent for rent, $\frac{1}{5}$ for clothes, $\frac{1}{4}$ for food, and $\frac{1}{2}$ of the remainder for travel. If the yearly income is $3000, how much is spent for travel?

15. A man sold $\frac{2}{3}$ of an acre of land for $76. At that rate what is his farm of 120 acres worth?

16. $\frac{4}{7}$ of the number of students in a high school are girls. The number of boys is 120. How many students are there in the school?

17. Mr. Jones bought some land and sold it so as to realize $\frac{1}{5}$ more than the cost. If the selling price was $360, what did he pay for the land?

18. Mr. Williams sold some goods for $\frac{1}{6}$ less than the cost. If he received $70 for the goods, what was the cost?

19. In making up a certain cake recipe for 6 people, $1\frac{1}{2}$ cups of sugar and $\frac{2}{3}$ of a cup of butter are used. How much sugar and butter should be used in making up the recipe for 4 people?

20. A boy works $3\frac{1}{2}$ days at the rate of $6.50 a week of 6 days. How much does he earn?

21. Find the total cost of $8\frac{2}{3}$ yards of ribbon at 28 cents per yard, $3\frac{1}{2}$ yards of insertion at $12\frac{1}{2}$ cents per yard, and $5\frac{1}{3}$ yards of silk at $\$1.35$ per yard.

22. A dealer bought oranges at the rate of 4 for 10 cents and sold them at the rate of 3 for 10 cents. How much did he gain on each orange?

23. Mr. Rankin owned $\frac{2}{3}$ of a store and sold $\frac{2}{3}$ of his share to Mr. Johnson for $\$3600$. At this rate what was the store worth?

24. If a man drives his automobile 18 miles in $\frac{2}{3}$ of an hour, how long would he require, at the same rate, to travel 60 miles?

25. When $\frac{3}{4}$ of a yard of cloth costs $\$1.80$, what is the price of $\frac{4}{5}$ of a yard?

26. A girl grew $1\frac{1}{4}$ inches during one year and $\frac{5}{8}$ of an inch during the next year. How much more did she grow during the first year than during the second?

27. The entire length of a skirt is to be $26\frac{3}{4}$ inches, and the ruffle at the bottom is to be $3\frac{7}{8}$ inches wide. What will be the length of the skirt above the ruffle?

28. The record for the hundred yard dash is $9\frac{3}{5}$ seconds. A boy can run the distance in $11\frac{4}{5}$ seconds. What is the difference between his time and the record time?

29. A farmer raised $296\frac{1}{3}$ bushels of wheat on $15\frac{1}{4}$ acres. What was the average yield per acre?

30. Three and one half bushels of seed were sown and the yield was $21\frac{1}{2}$ bushels. What was the average yield per bushel of seed?

Short Methods Involving Fractions

54. To add fractions when the numerators are the same.

Add the denominators and multiply this sum by the common numerator to obtain the numerator of the result. Multiply the denominators to obtain the denominator of the result. Reduce to simplest form.

Examples. 1. Add $\frac{1}{3}$ and $\frac{1}{7}$.

Solution. $3 + 7 = 10$, the numerator of the result.
$3 \times 7 = 21$, the denominator of the result.
Hence, $\frac{1}{3} + \frac{1}{7} = \frac{10}{21}$.

2. Add $\frac{2}{5}$ and $\frac{2}{9}$.

SOLUTION. $5 + 9 = 14$. $14 \times 2 = 28$, the numerator of the result.
$5 \times 9 = 45$, the denominator of the result.
Hence, $\frac{2}{5} + \frac{2}{9} = \frac{28}{45}$.

Oral Work

Add as indicated:

1. $\frac{1}{5} + \frac{1}{3}$.
2. $\frac{1}{7} + \frac{1}{9}$.
3. $\frac{1}{5} + \frac{1}{8}$.
4. $\frac{1}{9} + \frac{1}{16}$.
5. $\frac{1}{7} + \frac{1}{8}$.
6. $\frac{1}{12} + \frac{1}{7}$.
7. $\frac{1}{9} + \frac{1}{14}$.
8. $\frac{2}{7} + \frac{2}{13}$.
9. $\frac{1}{3} + \frac{1}{6}$.
10. $\frac{2}{3} + \frac{2}{8}$.
11. $\frac{2}{11} + \frac{2}{7}$.
12. $\frac{3}{5} + \frac{3}{7}$.
13. $\frac{3}{8} + \frac{3}{13}$.
14. $\frac{3}{5} + \frac{3}{11}$.

55. To subtract fractions when the numerators are the same.

Subtract the denominators and multiply this difference by the common numerator to form the numerator of the result. Multiply the denominators to form the denominator of the result. Reduce to simplest form.

Example. Subtract $\frac{2}{5}$ from $\frac{2}{3}$.

SOLUTION. $5 - 3 = 2$. $2 \times 2 = 4$, the numerator of the result.
$5 \times 3 = 15$, the denominator of the result.
Hence, $\frac{2}{3} - \frac{2}{5} = \frac{4}{15}$.

Oral Work

Perform the following subtractions:

1. $\frac{1}{6} - \frac{1}{8}$.
2. $\frac{1}{7} - \frac{1}{13}$.
3. $\frac{1}{8} - \frac{1}{12}$.
4. $\frac{2}{7} - \frac{2}{15}$.
5. $\frac{3}{7} - \frac{3}{16}$.
6. $\frac{5}{9} - \frac{5}{14}$.

56. To add two fractions by "cross multiplication."

Multiply the numerator of each fraction by the denominator of the other fraction. Add these products to form the numerator. Multiply the denominators to form the denominator. Reduce to lowest terms.

Example. Add $\frac{2}{3}$ and $\frac{3}{4}$.

SOLUTION. $2 \times 4 = 8$.
$3 \times 3 = 9$.
$9 + 8 = 17$, the numerator.
$3 \times 4 = 12$, the denominator.
Hence, $\frac{2}{3} + \frac{3}{4} = \frac{17}{12}$, $\frac{17}{12} = 1\frac{5}{12}$.

Oral Work

Add as indicated:

1. $\frac{3}{5} + \frac{2}{7}$.
2. $\frac{2}{3} + \frac{3}{7}$.
3. $\frac{1}{2} + \frac{3}{8}$.
4. $\frac{3}{10} + \frac{7}{16}$.
5. $\frac{8}{9} + \frac{7}{8}$.
6. $\frac{3}{5} + \frac{7}{8}$.
7. $\frac{5}{12} + \frac{3}{8}$.
8. $\frac{6}{7} + \frac{6}{11}$.
9. $\frac{5}{7} + \frac{9}{16}$.

57. To subtract fractions by "cross multiplication."

Multiply the numerator of each fraction by the denominator of the other fraction. Subtract these products to form the numerator. Multiply the denominators to form the denominator. Reduce to lowest terms.

Example. Subtract $\frac{2}{3}$ from $\frac{5}{6}$.

SOLUTION.

$5 \times 3 = 15.$
$6 \times 2 = 12.$
$15 - 12 = 3$, the numerator.
$6 \times 3 = 18$, the denominator.

Hence, $\frac{5}{6} - \frac{2}{3} = \frac{3}{18}$, $\frac{3}{18} = \frac{1}{6}$.

Written Work

Subtract as indicated:

1. $\frac{4}{7} - \frac{1}{3}$.
2. $\frac{8}{11} - \frac{3}{5}$.
3. $\frac{5}{8} - \frac{3}{10}$.
4. $\frac{7}{8} - \frac{7}{11}$.
5. $\frac{8}{15} - \frac{4}{11}$.
6. $\frac{13}{25} - \frac{4}{11}$.
7. $\frac{5}{3} - \frac{6}{17}$.

58. To find the approximate product of mixed numbers.

Multiply the integers. Multiply each integer by the other fraction to the nearest unit. Add these three products.

Example. Multiply $13\frac{1}{2}$ by $6\frac{2}{3}$.

SOLUTION.

$13\frac{1}{2}$
$6\frac{2}{3}$
$13 \times 6 = 78$
$\frac{1}{2}$ of $6 = 3$
$\frac{2}{3}$ of $13 = 9$ (to the nearest unit).
90

Written Work

Find approximate products:

1. $43\frac{1}{3}$ by $62\frac{1}{5}$
2. $89\frac{1}{4}$ by $7\frac{1}{2}$
3. $76\frac{1}{4}$ by $82\frac{1}{7}$
4. $15\frac{2}{5}$ by $40\frac{2}{7}$
5. $98\frac{1}{6}$ by $47\frac{1}{3}$

6. $17\frac{3}{8}$	**7.** $13\frac{2}{7}$	**8.** $79\frac{2}{9}$	**9.** $38\frac{2}{5}$	**10.** $73\frac{5}{8}$
$46\frac{4}{5}$	$16\frac{1}{4}$	$63\frac{1}{5}$	$71\frac{1}{6}$	$96\frac{7}{8}$

59. To find the product of any two numbers ending in $\frac{1}{2}$.

(*a*) When the sum of the integers is an even number.

To the product of the integers, add one half of their sum and annex $\frac{1}{4}$ to the result.

Example. Multiply $39\frac{1}{2}$ by $3\frac{1}{2}$.

SOLUTION.

$$\begin{aligned} &39\tfrac{1}{2} \\ &3\tfrac{1}{2} \\ 39 \times 3 &= 117 \\ \tfrac{1}{2} \text{ the sum of 39 and 3} &= 21 \\ &138\tfrac{1}{4} \quad \textit{Result.} \end{aligned}$$

Written Work

Multiply:

1. $44\frac{1}{2}$	**2.** $18\frac{1}{2}$	**3.** $38\frac{1}{2}$	**4.** $75\frac{1}{2}$	**5.** $29\frac{1}{2}$	**6.** $81\frac{1}{2}$
$64\frac{1}{2}$	$12\frac{1}{2}$	$14\frac{1}{2}$	$35\frac{1}{2}$	$33\frac{1}{2}$	$75\frac{1}{2}$

(*b*) When the sum of the integers is an odd number.

To the product of the integers, add the result obtained by taking half of one less than their sum. To this result annex $\frac{3}{4}$.

Example. Multiply $39\frac{1}{2}$ by $6\frac{1}{2}$.

SOLUTION.

$$\begin{array}{r} 39\frac{1}{2} \\ 6\frac{1}{2} \\ \hline 234 \\ 22 \\ \hline 256\frac{3}{4} \end{array}$$

Written Work

Multiply:

1. $73\frac{1}{2}$	**2.** $57\frac{1}{2}$	**3.** $92\frac{1}{2}$	**4.** $35\frac{1}{2}$	**5.** $98\frac{1}{2}$
$42\frac{1}{2}$	$24\frac{1}{2}$	$47\frac{1}{2}$	$58\frac{1}{2}$	$17\frac{1}{2}$
6. $29\frac{1}{2}$	**7.** $83\frac{1}{2}$	**8.** $76\frac{1}{2}$	**9.** $43\frac{1}{2}$	**10.** $98\frac{1}{2}$
$36\frac{1}{2}$	$96\frac{1}{2}$	$19\frac{1}{2}$	$46\frac{1}{2}$	$23\frac{1}{2}$
11. $48\frac{1}{2}$	**12.** $95\frac{1}{2}$	**13.** $73\frac{1}{2}$	**14.** $47\frac{1}{2}$	**15.** $27\frac{1}{2}$
$72\frac{1}{2}$	$81\frac{1}{2}$	$71\frac{1}{2}$	$33\frac{1}{2}$	$71\frac{1}{2}$

CHAPTER VIII

DECIMAL FRACTIONS

60. Comparison of Common and Decimal Fractions.

The denominator of a common fraction may be any number, and it is always expressed.

The denominator of a decimal fraction is always some power of 10, and the power is indicated by the number of figures to the right of the decimal point.

Thus, $.7 = \frac{7}{10}$. $.0007 = \frac{7}{10000}$.

$.07 = \frac{7}{100}$. $.423 = \frac{423}{1000}$.

$.007 = \frac{7}{1000}$. $.027 = \frac{27}{1000}$.

61. Reading and Writing Decimals.

The names of the various decimal orders are stated in the following table:

.	1	2	3	4	5	6	7
	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths	Ten-millionths

The names of the various orders should be learned. You should know that the *name* of the third decimal order is thousandths; that of the fifth is hundred-thousandths; that of the second is hundredths, etc.

To read a decimal fraction, read the number as if it were an integer and then state the name of the last decimal order which a digit of the number occupies.

Pronounce the word "and" at the decimal point and omit it elsewhere, except in reading complex decimals as noted below:

.045 is read forty-five thousandths.

.505 is read five hundred five thousandths.

500.005 is read five hundred *and* five thousandths.

.4206 is read four thousand two hundred six ten-thousandths.

.0$\frac{2}{3}$ is read two thirds of a tenth.

2.0$\frac{2}{3}$ is read two *and* two thirds of a tenth.

.2$\frac{2}{3}$ is read two and two thirds tenths (a complex decimal).

Read:

1. .043.	2. .1764.	3. 2.04.
4. 500.17.	5. 400.004.	6. .00$\frac{2}{3}$.
7. 214.0014.	8. .000562.	9. 64.3.
10. 14.003.	11. .003726.	12. 542.54.
13. 3005.016.	14. .000241$\frac{1}{3}$.	15. 7.00707.

Write decimally:

1. Seven tenths; forty-two thousandths; five hundred four thousandths.

2. Four hundred and three thousandths; eight hundred two ten-thousandths.

3. Three fourths of a tenth; two and one seventh tenths.

4. Twenty-four ten-thousandths; five millionths; seventeen hundred-thousandths; two hundred and three hundredths.

5. Five hundred seventeen millionths; sixteen ten-thousandths; eighteen hundred and fifteen ten-thousandths.

6. Five thousand four hundred-millionths; eighteen and four thousandths.

62. Addition of Decimal Fractions.

Place the decimals so that the decimal points fall vertically. Add as with integers. Place the decimal point of the sum directly below those in the numbers added.

Example. Find the sum of 4.023, .507, 2.003, and 1.125.

SOLUTION.

$$\begin{array}{r} 4.023 \\ .507 \\ 2.003 \\ 1.125 \\ \hline 7.658 \end{array}$$

Written Work

1. Add 2569.327, 1462.978, 47.9634, 2693.072, and .019.
2. Add 27.9548, 91.0005, 37.427, and 27563.974.
3. Add 2752.9374, .0003, 23.247, and 259.6347.

Change the following common fractions to decimal fractions, and add:

4. $\frac{24}{100}, \frac{372}{1000}, \frac{19}{10}, \frac{254}{100}$.
5. $\frac{3}{1000}, \frac{24}{10000}, \frac{675}{10}, \frac{170}{10000}$.
6. $\frac{36.5479}{100}, \frac{762}{1000}, \frac{7642}{10000}, \frac{271}{1000}$.
7. Copy, find the totals required, and check:

\$ 324.80	\$ 3764.20	\$7436.08	?
17.04	762.83	427.36	?
9320.87	9402.03	92.18	?
473.26	1.18	4724.63	?
791.14	13420.03	924.56	?
?	?	?	?

63. Subtraction of Decimal Fractions.

Place the decimals so that the points fall vertically. Subtract as with integers, then place the decimal point in the remainder immediately below the decimal points above.

Example. Subtract 23.213 from 34.047.

SOLUTION.

$$\begin{array}{r} 34.047 \\ 23.213 \\ \hline 10.834 \end{array}$$

Subtract as indicated. Check the results.

1. $394.237 - 1.027$. 2. $.47 - .0003$. 3. $.394237 - .15$.

4. $7763.421 - 28.796$. 5. $87.5932 \quad 2.3579$.

Change to decimal fractions and subtract:

6. $\frac{465}{1000} - \frac{23}{100}$. 7. $\frac{76943}{100} - \frac{235}{1000}$. 8. $\frac{8732}{10} - \frac{236547}{100000}$.

9. The sum of two numbers is 342.086. One of the numbers is 206.78. What is the other number?

10. A man deposited \$5764.80 in a bank. He later drew out \$3780.92 and then he deposited \$4814.60. How much had he then?

64. Multiplication of Decimal Fractions.

Multiply as with integers. From the right of the product, point off as many places as the sum of the number of decimal places in the multiplicand and the multiplier.

Examples. 1. Multiply 4.625 by .05.

SOLUTION.

$$\begin{array}{r} 4.625 \\ \times \quad .05 \\ \hline .23125 \end{array}$$

2. Multiply .00362 by .06.

SOLUTION.

$$\begin{array}{r} .00362 \\ \times \quad .06 \\ \hline .0002172 \end{array}$$

Written Work

Multiply as indicated. Check the results.

1. $25.763 \times .1463$. 2. $75.46 \times .03$.

3. $.2462 \times .347$. 4. $.083 \times .0462$.

5. $78.23 \times .000007$. 6. $17.13 \times .042$.

7. $570.04 \times .00326$. 8. $\$374.26 \times .055$.

9. $\$9037.28 \times .035$. 10. $\$473.54 \times .335$.

11. Find the cost of 46 lots at an average cost of \$847.60.

12. Find the cost of 143 barrels of apples at an average price of \$3.64 per barrel.

65. Division of Decimal Fractions.

a. When the divisor is an integer.

Examples. 1. $.12 \div 4 = .03$; just as $\$12 \div 4 = \$3$.

2. $.426 \div 3 = .142$; just as $\$426 \div 3 = \$142$.

In making the division the decimal point should be written in the quotient when it is reached in the dividend.

3. Divide 157.25 by 37.

SOLUTION.

$$
\begin{array}{r}
4.25 \\
37\overline{)157.25} \\
\underline{148} \\
9.2 \\
\underline{7.4} \\
1.85 \\
\underline{1.85}
\end{array}
$$

b. When the divisor is not an integer.

Multiply both dividend and divisor by the least power of ten that will make the divisor an integer. Divide as in case *a*.

Example. Divide .028 by .4.

SOLUTION. First multiply both dividend and divisor by 10, to make the divisor an integer. Then divide the resulting dividend, .28, by 4. The result is .07.

Similarly, $.0456 \div .03 = 4.56 \div 3 = 1.52$.
Similarly, $5.64 \div .0004 = 56400 \div 4 = 14100$.

If there is a remainder after all of the decimal places in the dividend have been used, zeros may be annexed to the dividend and the division may be carried as far as is desired.

Oral Work

1. .08 by .4.	2. .014 by .2.	3. .364 by .04.
4. 3300 by .11.	5. 8.48 by .008.	6. .220 by .0011.
7. 1 by .01.	8. .10 by .1.	9. .018 by 18.
10. .0001 by .00001.	11. .042 by 2.1.	12. 1.1 by .011.

Written Work

1. $268632 \div 36.9$.
2. $26863.2 \div 36.9$.
3. $26863.2 \div 369$.
4. $2686.32 \div 36.9$.
5. $26.8632 \div 36.9$.
6. $2686.32 \div .369$.
7. $26.8632 \div .369$.
8. $26.8632 \div .0369$.
9. $2.68632 \div .369$.
10. $2.68632 \div 3.69$.
11. $2.68632 \div 36.9$.
12. $2.68632 \div 369$.
13. $.268632 \div 369$.
14. $2.68632 \div 3690$.
15. $293.45 \div 14.24$.
16. $2.734 \div 1.32$.
17. $.73469 \div 127.9638$.
18. $762.397 \div 36947.28$.
19. $.1479376 \div 293.4798$.
20. $.0097 \div 12.34692$.

21. The multiplier is .045, the product is .01665. What is the multiplicand?

22. At \$.24 per dozen, how many eggs can be bought for \$4.08?

23. If a man's annual income is \$5420 and his annual expenses are \$4262, what are his average weekly savings?

24. What number is $\frac{3}{7}$ as large as .0427?

25. What number divided by 4.28 gives a quotient of .07 and a remainder of .04?

26. In a certain factory 30 men are employed at \$2.15 per day, 10 men at \$3.60 per day, 28 men at \$2.90 a day, and 12 men at \$1.80 a day. Find the average daily wage.

27. Using the current market price find the cost of the following: 1 pk. of apples, $\frac{1}{2}$ bu. of sweet potatoes, $\frac{1}{2}$ lb. of tea, 6 bars of laundry soap, $1\frac{1}{2}$ lb. porterhouse steak.

28. A train ran at the rate of .87 mile a minute. At this rate how many miles would it travel in $3\frac{1}{2}$ minutes? In 10 minutes? In 1 hour?

29. An automobile traveled 72.25 miles in $2\frac{1}{3}$ hours. What was the average rate of speed per hour?

66. Changing Decimal Fractions to Equivalent Common Fractions.

Omit the decimal point; write for the denominator 1 *with as many zeros annexed as there are places in the decimal. Reduce to lowest terms.*

Examples. 1. Change .75 to a common fraction.

SOLUTION. $.75 = \frac{75}{100} = \frac{3}{4}$.

2. Change .00864 to a common fraction.

SOLUTION. $.00864 = \frac{864}{100000} = \frac{27}{3125}$.

Written Work

Change the following decimals to equivalent common fractions and reduce to lowest terms:

1. .48.	2. .095.	3. .3705.	4. .0012.	5. 3.6425.
6. .125.	7. .9825.	8. .625.	9. .1875.	10. .0015.
11. $.06\frac{1}{4}$.	12. $.002\frac{1}{2}$.	13. $.37\frac{1}{2}$.	14. .333.	15. $.012\frac{1}{2}$.
16. $.006\frac{2}{3}$.	17. .00625.	18. .0036.	19. $.008\frac{1}{3}$.	20. $.009\frac{1}{11}$.
21. $.001\frac{1}{4}$.				

67. Changing Common Fractions to Equivalent Decimal Fractions.

Since a common fraction may be regarded as an indicated division, the reduction may be made by the methods of division previously explained.

Example. Change $\frac{4}{5}$ to a decimal fraction.

SOLUTION. $\frac{4}{5} = \frac{1}{5}$ of 4. Place a decimal point to the right of the 4, annex zeros, and divide.

$$5\overline{)4.0}\quad .8$$

Similarly, $\frac{7}{8} = \frac{1}{8}$ of 7.

$$8\overline{)7.000}\quad .875$$

Similarly, $\frac{2}{375} = \frac{1}{375}$ of 2.

$$375\overline{)2.000}\quad .0053+$$

$$1.875$$

$$.1250$$

$$.1125$$

$$125$$

It is usually not necessary to carry the division more than three or four places.

For example, $\frac{1}{7}$ may be expressed decimally as $.142\frac{6}{7}$ or as $.1429^{-}$.

Written Work

Considering the figures in the top row as numerators, and those at the left as denominators, change the common fractions thus formed to decimals, entering the results in the proper places. Carry the results to three decimal places.

Prepare the work on a carefully ruled form.

68. Approximate Results. For most practical purposes it is unnecessary to carry a result to more than three decimal places. Thus, $25.86347 is stated with sufficient accuracy as $25.86.

In finding approximate decimal fractions, follow these directions.

Carry the division at least one more place than the number of digits desired in the result. If the digit at the right of the last place desired in the result is less than 5, drop it; if it is 5 or larger than 5, increase the preceding digit by 1.

Denominators	Numerators								
	3	7	6	26	47	126	79	814	1216
18									
27									
94									
38									
172									
9									
66									
213									
793									
2361									
13									
109									

Written Work

The following table gives the land area, in acres, of the different geographical sections of the United States, and the number of acres in each section devoted to farming.

Prepare a ruled form similar to the model.

Find what decimal part of the land area of each section, and of the United States, is improved farm land. (Approximate results to the nearest thousandth.)

Also find the total land area of the United States, the total area of the improved land, and the fraction of the total area which is improved.

Arrange the different sections on the blank in the order of their rank, placing the section with the largest fraction of improved farm land at the top of the blank.

Section	Total Land Area	Improved Farm Land	
	Acres	Acres	Decimal Fraction of Total
New England	39,664,640	7,254,904	
Middle Atlantic	64,000,000	29,320,894	
East North Central	157,160,960	88,947,228	
West North Central	326,914,560	164,284,862	
South Atlantic	172,205,440	48,479,733	
East South Central	114,885,760	43,946,846	
West South Central	275,037,440	58,264,273	
Mountain	549,840,000	15,915,002	
Pacific	203,580,800	22,038,008	
United States			

Oral Work

The following table shows :

The value of the butter *sold* in each section of the United States in a recent year.

The fractions which these values were of the *total production* of butter in each section.

VALUE OF BUTTER PRODUCED IN UNITED STATES

SECTION	VALUE OF BUTTER SOLD	FRACTION OF TOTAL PRODUCT SOLD	VALUE OF TOTAL PRODUCTION
New England	$ 8,533,864	.725	$
Middle Atlantic . . .	15,229,862	.655	
East North Central . .	31,855,809	.585	
West North Central . .	20,333,127	.438	
South Atlantic	7,622,916	.275	
East South Central . .	4,842,959	.167	
West South Central . .	5,381,690	.19	
Mountain	2,166,918	.422	
Pacific	4,410,978	.572	

1. By referring to the table we find that .725 of the butter produced in New England was sold. How many tenths of the butter made in New England was sold?

2. If $4,410,978 is about fifty-seven hundredths of the value of all the butter made in the Pacific States in a year, how would you find the total value of the butter production in this section?

NOTE. When in doubt whether the solution of a given problem requires multiplication or division, replace the given numbers by smaller numbers. Reread the problem with these small numbers, and decide upon the process.

3. Will the numbers to be recorded in the last column of the preceding blank be larger or smaller than those in the first column?

Written Work

Rule a blank similar to the one above.

Find the total value of the butter produced in each section. (Approximate results to the nearest dollar.)

Enter the sections on the blank in the order of their rank as butter producers.

Review of Decimal Fractions

Written Work

1. In a recent year 1,591,311,371 dozen eggs were produced in the United States. The following decimals show the fractional part of this number produced in the different geographical sections.

New England	.035
Middle Atlantic	.102
East North Central	.247
West North Central	.28
South Atlantic	.086
East South Central	.081
West South Central	.104
Mountain	.022
Pacific	.043

Find the number of dozen eggs produced in each section, and tabulate this on a ruled form. How can you check the accuracy of your work?

2. The value of the wool produced by the mountain states in 1909 was .608 more than the value of the wool produced by these states in 1899. The increase in dollars was $11,039,843. What was the value in 1899? In 1909?

3. The mountain states include a large portion of the grazing land of the country and for this reason produce a large part of the wool of the country. In 1909 they produced .4332 of the total value of the wool grown in the United States. By referring to your results in the preceding problem, find the total value of the wool grown in the United States in 1909.

4. In 1909 Wyoming produced the largest value of wool grown in any state of the Union. The value of its product was $8,912,608. This was what decimal part of the value of the wool product of the entire country for the year?

5. Montana, the second largest producer of wool, raised wool valued at $8,223,754. Ohio, the third largest producer, raised

wool valued at $6,749,005. Fill in the blanks in the following sentences with the proper decimal fractions:

Wyoming produced —— times as much wool as Montana.
Ohio produced —— as much wool as Wyoming.

6. If a boy's average gain in weight is 8.25 lb. per year, how much will he gain in weight in $2\frac{1}{2}$ yr.?

7. If one turn of a screw advances the point .14 in., how far will 7 turns advance it?

8. A gallon of milk weighs 8.622 lb. and a gallon of water weighs 8.355 lb. How much more does $\frac{1}{2}$ gal. of milk weigh than $\frac{1}{2}$ gal. of water?

9. A boy can run 100 yd. in 11.2 sec. At this rate, how long would it take him to run 80 yd.?

10. In a certain city the rainfall during March was 2.46 in., during April 3.15 in., and during May 1.09 in. Find the average for these months.

11. A man sold 24 dozen eggs, which was .4 of all that he had. How many did he have?

12. If .375 of an acre of land sells for $48, what should $2\frac{1}{2}$ acres sell for, at the same rate?

13. An English pound contains 113.00001 grains of pure gold, and an American gold dollar contains 23.22 grains of pure gold. Find the value of an English pound in American money.

CHAPTER IX

SHORT METHODS INVOLVING ALIQUOT PARTS

69. Equivalent Common and Decimal Fractions in Frequent Use.

$.50 = \frac{1}{2}$.	$.12\frac{1}{2} = \frac{1}{8}$.	$.06\frac{1}{4} = \frac{1}{16}$.	$.62\frac{1}{2} = \frac{5}{8}$.
$.33\frac{1}{3} = \frac{1}{3}$.	$.11\frac{1}{9} = \frac{1}{9}$.	$.05 = \frac{1}{20}$.	$.75 = \frac{3}{4}$.
$.25 = \frac{1}{4}$.	$.10 = \frac{1}{10}$.	$.40 = \frac{2}{5}$.	$.80 = \frac{4}{5}$.
$.20 = \frac{1}{5}$.	$.09\frac{1}{11} = \frac{1}{11}$.	$.37\frac{1}{2} = \frac{3}{8}$.	$.83\frac{1}{3} = \frac{5}{6}$.
$.16\frac{2}{3} = \frac{1}{6}$.	$.08\frac{1}{3} = \frac{1}{12}$.	$.60 = \frac{3}{5}$.	$.87\frac{1}{2} = \frac{7}{8}$.
$.14\frac{2}{7} = \frac{1}{7}$.	$.06\frac{2}{3} = \frac{1}{15}$.		

Commit these common fractions and their equivalent decimals to memory, as they can be used to shorten many computations.

70. A Short Method of Multiplying by These Fractions.

Which is easier, $1248 \times .16\frac{2}{3} = 208$; or $\frac{1}{6}$ of $1248 = 208$?

State a short method of multiplying by each of the decimal fractions in the table above.

Oral Work

Multiply:

1. $642 \times .50$.
2. $936 \times .33\frac{1}{3}$.
3. $488 \times .25$.
4. $650 \times .20$.
5. $366 \times .16\frac{2}{3}$.
6. $840 \times .14\frac{2}{7}$.
7. $720 \times .12\frac{1}{2}$.
8. $456 \times .08\frac{1}{3}$.
9. $930 \times .06\frac{2}{3}$.
10. $320 \times .06\frac{1}{4}$.

The results in the preceding exercise are integers. The same method may be used when the division results in a fraction.

Example. Multiply 4634 by .25.

SOLUTION. $.25 = \frac{1}{4}$.

$4634 \div 4 = 1158.5$.

Oral Work

Without copying, find the results of the following:

1. $8735 \times .50$.
2. $3940 \times .33\frac{1}{3}$.
3. $267 \times .25$.
4. $379.3 \times .16\frac{2}{3}$.
5. $842 \times .20$.
6. $3824 \times .14\frac{2}{7}$.
7. $5316 \times .12\frac{1}{2}$.
8. $4026 \times .08\frac{1}{3}$.
9. $1848 \times .06\frac{2}{3}$.
10. $1940 \times .06\frac{1}{4}$.
11. $35.69 \times .33\frac{1}{3}$.
12. $649.3 \times .25$.
13. $.8347 \times .50$.
14. $723.68 \times .20$.
15. $36.92 \times .14\frac{2}{7}$.
16. $56.32 \times .12\frac{1}{2}$.
17. $7836 \times .06\frac{1}{4}$.
18. $1223 \times .16\frac{2}{3}$.
19. $3380 \times .06\frac{2}{3}$.

20. Find the value of each of the following:

48 lb. at $37\frac{1}{2}\,¢$ per pound.
30 yd. at $62\frac{1}{2}\,¢$ per yard.
32 lb. at $12\frac{1}{2}\,¢$ per pound.
21 lb. at $14\frac{2}{7}\,¢$ per pound.
21 lb. at $87\frac{1}{2}\,¢$ per pound.
48 yd. at $6\frac{2}{3}\,¢$ per yard.
24 yd. at $12\frac{1}{2}\,¢$ per yard.
96 lb. at $83\frac{1}{3}\,¢$ per pound.

71. A Short Method of Dividing by These Fractions.

Which is easier, $6 \div .25 = 24$; or $6 \times 4 = 24$?

State a short method of dividing by each of the decimal fractions in the table on page 85.

Oral Work

Without copying, state the results of the following:

1. $7 \div .12\frac{1}{2}$.
2. $49 \div .50$.
3. $126 \div .33\frac{1}{3}$.
4. $63 \div .14\frac{2}{7}$.
5. $8 \div .06\frac{1}{4}$.
6. $10 \div .06\frac{2}{3}$.
7. $15 \div .25$.
8. $1627 \div .20$.
9. $13.63 \div .16\frac{2}{3}$.
10. $1.43 \div .25$.
11. $3.249 \div .50$.
12. $.023 \div .08\frac{1}{3}$.
13. $63.98 \div .2$.
14. $.043 \div .16\frac{2}{3}$.

72. Aliquot Parts which are not Unit Fractions.

State a short method of multiplying by $.37\frac{1}{2}$; by $.66\frac{2}{3}$; by .75; by $.87\frac{1}{2}$.

State a short method of dividing by $.37\frac{1}{2}$; by $.66\frac{2}{3}$; by .75; by $.87\frac{1}{2}$.

When dividing by one number and multiplying by another, observe the following directions:

Divide first if the division results in an integer. If the division would result in a mixed number, multiply first.

Examples. 1. Multiply 32 by $.37\frac{1}{2}$.

SOLUTION. $.37\frac{1}{2} = \frac{3}{8}$.

$32 \div 8 = 4$.

$4 \times 3 = 12$. Since 8 is a factor of 32, the division is performed first.

2. Divide 16 by $.37\frac{1}{2}$.

SOLUTION. Substituting the equivalent common fraction, we have

$16 \div \frac{3}{8} = ?$

or $16 \times \frac{8}{3} = ?$ Inverting the divisor.

$16 \times 8 = 128$.

$128 \div 3 = 42\frac{2}{3}$. Since 3 is not a factor of 16, the multiplication is performed first.

What advantage is there in observing these directions?

Written Work

Without recopying, find the results of the following:

1. $64 \times .375$.	**2.** $96 \times .66\frac{2}{3}$.	**3.** $480 \times .75$.
4. $168 \times .875$.	**5.** $248 \times .87\frac{1}{2}$.	**6.** $3260 \times .75$.
7. $4864 \times .37\frac{1}{2}$.	**8.** $639 \times .66\frac{2}{3}$.	**9.** $42 \div .37\frac{1}{2}$.
10. $56 \div .87\frac{1}{2}$.	**11.** $40 \div .66\frac{2}{3}$.	**12.** $36 \div .75$.
13. $1462 \div .66\frac{2}{3}$.	**14.** $465 \div .375$.	**15.** $11322 \div .75$.
16. $994 \div .875$.	**17.** $426 \times .875$.	**18.** $143 \times .37\frac{1}{2}$.
19. $1347 \times .75$.	**20.** $539 \times .66\frac{2}{3}$.	**21.** $3.29 \times .75$.
22. $1.267 \times .875$.	**23.** $139 \div .87\frac{1}{2}$.	**24.** $1426 \div .37\frac{1}{2}$.
25. $139 \div .66\frac{2}{3}$.	**26.** $46 \div .75$.	**27.** $46.9 \div .37\frac{1}{2}$.
28. $.429 \div .66\frac{2}{3}$.		

73. General Table of Aliquot Parts. The following table shows the most frequently used decimal parts of 1, 10, 100, and 1000.

Fractions	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{3}{8}$
of 1	.50	.33$\frac{1}{3}$	.66$\frac{2}{3}$	.25	.75	.125	.375
of 10	5.	3.33$\frac{1}{3}$	6.66$\frac{2}{3}$	2.5	7.5	1.25	3.75
of 100	50.	33.33$\frac{1}{3}$	66.66$\frac{2}{3}$	25.	75.	12.5	37.5
of 1000	500.	333$\frac{1}{3}$	666$\frac{2}{3}$	250.	750.	125.	375.

Fractions	$\frac{5}{8}$	$\frac{5}{6}$	$\frac{7}{8}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{16}$	$\frac{1}{6}$	$\frac{1}{7}$
of 1	.625	.83$\frac{1}{3}$	.875	.08$\frac{1}{3}$	.06$\frac{2}{3}$	.06$\frac{1}{4}$	.16$\frac{2}{3}$	.14$\frac{2}{7}$
of 10	6.25	8.33$\frac{1}{3}$	8.75	.82$\frac{1}{2}$		.625	1.6$\frac{2}{3}$	1.42
of 100	62.5	83.3$\frac{1}{3}$	87.5	8.3$\frac{1}{3}$		6.25	16$\frac{2}{3}$	14.28
of 1000	625.	833.3$\frac{1}{3}$	875.	83$\frac{1}{3}$		62.5	166$\frac{2}{3}$	142.8

Study this table until you can recognize quickly these fractional parts of 1, 10, 100, and 1000.

74. Rules for Multiplication by Aliquot Parts of 1, 10, 100, and 1000.

a. To multiply by fractional parts of 10.

Multiply by the equivalent fraction; multiply this product by 10, *by moving the decimal point one place to the right, annexing a zero, if necessary.*

Examples. 1. Multiply 16 by $2\frac{1}{2}$.

Solution. $2\frac{1}{2}$ is $\frac{1}{4}$ of 10. $\frac{1}{4}$ of $16 = 4$. Multiply by 10, by annexing a zero; the result is 40.

2. Multiply 26 by 1.25.

Solution. 1.25 is $\frac{1}{8}$ of 10. $\frac{1}{8}$ of $26 = 3.25$. Move the decimal point one place to the right; the result is 32.5.

b. To multiply by fractional parts of 100.

Multiply by the equivalent common fraction; multiply this product by 100.

Examples. 1. Multiply 24 by $87\frac{1}{2}$.

SOLUTION. $87\frac{1}{2}$ is $\frac{7}{8}$ of 100.
$\frac{7}{8}$ of $24 = 21$.
$21 \times 100 = 2100$.

2. Multiply 4252 by 12.5.

SOLUTION. 12.5 is $\frac{1}{8}$ of 100.
$\frac{1}{8}$ of $4252 = 531.5$.
$531.5 \times 100 = 53{,}150$.

Oral Work

State a short method of multiplying by each of the following:

$.06\frac{1}{4}$;	2.50;	$6\frac{2}{3}$;	$33\frac{1}{3}$;	250;	$87\frac{1}{2}$;	6.25;
$37\frac{1}{2}$;	7.50;	5;	$333\frac{1}{3}$;	75;	125;	$8\frac{1}{3}$;
$2\frac{1}{2}$;	12.50;	$6\frac{1}{4}$;	$62\frac{1}{2}$;	$1\frac{1}{4}$;	$8\frac{3}{4}$;	25;
$66\frac{2}{3}$;	375;	750;	875;	37.50;	7500;	12,500.

Find the products of the following. Do the work orally when possible. Be prepared to explain how the short method was applied in each example.

1. 32×25.	2. $111 \times 3\frac{1}{3}$.	3. $390 \times 66\frac{2}{3}$.
4. 648×250.	5. 724×75.	6. $832 \times 1\frac{1}{8}$.
7. $96 \times 87\frac{1}{2}$.	8. $108 \times 83\frac{1}{3}$.	9. $3264 \times .625$.
10. $144 \times 8\frac{1}{3}$.	11. 80×875.	12. 48×375.
13. 464×12.50.	14. $24 \times 7\frac{1}{2}$.	15. 3.20×25.
16. $.27 \times 6\frac{2}{3}$.	17. 8.4×3.75.	18. 640×6.25.
19. $3.612 \times .83\frac{1}{3}$.	20. $208 \times 3\frac{3}{4}$.	21. 143×250.
22. 1936×62.50.	23. 1721×7.50.	24. 1239×125.
25. 426×750.		

75. Interchanging Multiplicand and Multiplier. When the multiplicand and multiplier are abstract numbers, they may be interchanged.

Example. Multiply 25 by 428.

SOLUTION. Interchanging, 428×25
$\frac{1}{4}$ of $428 = 107$.
$107 \times 100 = 10{,}700$.

Written Work

Perform the following multiplications as indicated. Explain how the short method was applied in each example.

1. 924×12.50.	2. 864×750.	3. 125×488.
4. $264 \times .12\frac{1}{2}$.	5. $45 \times .06\frac{2}{3}$.	6. 250×3288.
7. 62.50×648.	8. $8\frac{1}{3} \times 3.60$.	9. 342×87.50.
10. 375×112.	11. $6\frac{2}{3} \times 456$.	12. 1468×250.
13. 875×88.	14. 1250×96.	15. $1776 \times .625$.
16. $7.2 \times 8\frac{3}{4}$.	17. $1\frac{1}{4} \times 104$.	18. 125×1.248.
19. 66×875.	20. $83\frac{1}{3} \times 9.6$.	21. $62\frac{1}{2} \times 20$.

Find the cost of the following:

22.
12 articles at \$.50 each
16 articles at .25 each
18 articles at .33$\frac{1}{3}$ each
30 articles at .40 each
16 articles at .75 each
Total ____

23.
64 doz. articles at \$ .62$\frac{1}{2}$ per dozen
76 doz. articles at 8.75 per dozen
58 doz. articles at .75 per dozen
125 doz. articles at .16 per dozen
6$\frac{1}{4}$ doz. articles at .32 per dozen
Total ____

24.
72 lb. at \$ .83$\frac{1}{3}$ per pound
52 lb. at 1.25 per pound
44 lb. at .37$\frac{1}{2}$ per pound
90 lb. at .12$\frac{1}{2}$ per pound
112 lb. at .06$\frac{1}{4}$ per pound
Total ____

25.
80 yd. at \$ .62$\frac{1}{2}$ per yard
66$\frac{2}{3}$ yd. at 1.20 per yard
86 yd. at 1.25 per yard
25 yd. at .78 per yard
75 yd. at .66$\frac{2}{3}$ per yard
Total ____

26. $83\frac{1}{3}$ yd. at \$1.80 per yard
$12\frac{1}{2}$ doz. at 1.60 per dozen
$8\frac{3}{4}$ lb. at 1.60 per pound
$6\frac{2}{3}$ yd. at 2.10 per yard
$3\frac{1}{3}$ yd. at 1.80 per yard
Total ____

27. 135 yd. at \$.$06\frac{1}{4}$ per yard
56 yd. at .$08\frac{3}{4}$ per yard
96 yd. at .$16\frac{2}{3}$ per yard
152 yd. at .$12\frac{1}{2}$ per yard
24 yd. at .$07\frac{1}{2}$ per yard
Total ____

76. Rules for Division by Aliquot Parts of 1, 10, 100, 1000.

a. To divide by fractional parts of 1.

Divide by the equivalent common fraction.

Example. Divide 13 by .$12\frac{1}{2}$. SOLUTION. $.12\frac{1}{2} = \frac{1}{8}$.
$13 \div \frac{1}{8} = 13 \times \frac{8}{1} = 104.$

b. To divide by fractional parts of 10.

Divide by 10; *divide this quotient by the equivalent common fraction.*

Example. Divide 80 by 2.50. SOLUTION. 2.50 is $\frac{1}{4}$ of 10.
$80 \div 10 = 8.$
$8 \div \frac{1}{4} = 8 \times \frac{4}{1} = 32.$

c. To divide by fractional parts of 100.

Divide by 100; *divide this quotient by the equivalent common fraction*

Oral Work

1. State a rule for dividing by fractional parts of 1000.
2. \$100 will pay for how many articles at \$.$33\frac{1}{3}$ each?
3. \$12 will pay for how many yards of cloth at \$.25 per yard?
4. Which is easier —

$$14 \div .75 = ? \qquad \text{or} \qquad 14 \div \tfrac{3}{4} = ?$$

5. State how to find the total cost of some articles, when the number of articles bought is given and the cost of each is $8\frac{1}{3}$ ¢; $14\frac{2}{7}$ ¢; $87\frac{1}{2}$ ¢; $16\frac{2}{3}$ ¢; $33\frac{1}{3}$ ¢; $62\frac{1}{2}$ ¢; $37\frac{1}{2}$ ¢.

Without copying, state results for the following examples. State the method of operation in each.

6. $15 \div .12\frac{1}{2}$.
7. $14 \div .06\frac{1}{4}$.
8. $9 \div .25$.
9. $\$6 \div \$.75$.
10. $\$14 \div \$.87\frac{1}{2}$.
11. $\$8 \div \$1.33\frac{1}{3}$.

State the quotients of each of the following:

	Dividend	Divisor	Quotient		Dividend	Divisor	Quotient
12.	42	$.33\frac{1}{3}$		13.	13	.25	
14.	22	$.66\frac{2}{3}$		15.	36	.75	
16.	5	$.06\frac{1}{4}$		17.	45	$.37\frac{1}{2}$	
18.	17	.125		19.	28	87.5	
20.	15	1.25		21.	900	6.25	
22.	400	$8.33\frac{1}{3}$		23.	150	$.06\frac{2}{3}$	

Written Work

Complete the following:

	Total Cost	Price Each	Number Purchased
1.	$ 5.25	$ $.37\frac{1}{2}$	
2.	20.25	.75	
3.	56.00	$.66\frac{2}{3}$	
4.	40.00	1.25	
5.	77.00	$.87\frac{1}{2}$	
6.	212.50	2.50	
7.	97.50	3.75	

8. Divide each of the following by $.12\frac{1}{2}$:

$.37\frac{1}{2}$; $.33\frac{1}{3}$; .625; $.87\frac{1}{2}$; .25; $1.14\frac{2}{7}$.

9. Find the sum of $\frac{1}{4}$, $\frac{1}{5}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, and $\frac{1}{2}$ and express the sum decimally.

10. Divide 36 by each of the following:

$.37\frac{1}{2}$; $.12\frac{1}{8}$; $.14\frac{2}{7}$; .50; $.06\frac{1}{4}$.

11. How many yards can be bought in each case :

Total Cost	Cost per Yard	Number of Yards
\$16	\$ .25	
24	$.33\frac{1}{3}$	
36	$.37\frac{1}{2}$	
20	$.62\frac{1}{2}$	
32	.50	
26	$.66\frac{2}{3}$	
56	1.25	
44	$1.33\frac{1}{3}$	
24	$2.66\frac{2}{3}$	

Review of Short Methods

Perform the following operations, telling what short method was used in each case:

1. $360 \div .37\frac{1}{2}$.	**2.** $\frac{2}{7} + \frac{3}{8}$.	**3.** 436×11.
4. 65×25.	**5.** 463×287.	**6.** 1016×1005.
7. 3654×2700.	**8.** 105×126.	**9.** 95×87.
10. $\frac{1}{3} - \frac{1}{5}$.	**11.** $48\frac{1}{4} \times 63\frac{7}{8}$.	**12.** $61\frac{1}{2} \times 12\frac{1}{2}$.
13. 109×113.	**14.** $\frac{9}{16} + \frac{8}{11}$.	**15.** 463×416.
16. 96×94.	**17.** 1015×1014.	**18.** 13×18.
19. $6\frac{1}{2} \times 4\frac{1}{2}$.	**20.** 825×927.	**21.** $.000825 \times 100$.
22. 17×12.	**23.** 724×999.	**24.** 116×104.
25. $\frac{3}{5} \div \frac{7}{8}$.	**26.** 113×102.	**27.** 1026×1003.
28. $\frac{1}{5} + \frac{1}{3}$.	**29.** 5362×111.	**30.** 433×99.
31. 109×112.	**32.** 976×990.	**33.** $33\frac{1}{2} \times 12\frac{1}{2}$.
34. 17×19.	**35.** $13\frac{1}{2} \times 14\frac{2}{5}$.	**36.** $\frac{3}{7} + \frac{3}{8}$.
37. $\frac{3}{7} - \frac{3}{8}$.	**38.** $15\frac{1}{2} \times 18\frac{1}{2}$.	**39.** 96×88.
40. 905.387×3000.	**41.** 986×993.	**42.** 16×18.
43. 874×1236.		

CHAPTER X

DENOMINATE NUMBERS

Tables of Weights and Measures

77. Long Measure.

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	= 1 rod (rd.)
320 rods, or 5280 feet	= 1 mile (mi.)
1760 yards	= 1 mile

Architects, carpenters, and mechanics frequently write ′ for foot, and ″ for inch. Thus 8′ 7″ means 8 ft. 7 in.

In engineering it is customary to divide the foot and the inch into tenths and hundredths, instead of into halves and fourths. There is a growing tendency to use the decimal division of the units.

Other measures of length are:

1 hand = 4 in. Used in measuring the height of horses.
1 fathom = 6 ft. Used in measuring depths at sea.
1 knot, nautical or geographical mile = 1.152$\frac{2}{3}$ mi. or 6086 ft.

The knot is used in measuring distances at sea. It is equivalent to 1 min. of longitude at the equator.

78. Square Measure.

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

Sq. ′ and sq. ″ are frequently used for square foot and square inch. Thus, 15 sq. ′ 6 sq. ″ means 15 sq. ft. 6 sq. in.

A *square* is 100 sq. ft. It is used in measuring roofing.

79. Cubic Measure.

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
128 cubic feet	= 1 cord

A cubic yard (of earth) is considered a load.

$24\frac{3}{4}$ cubic feet	= 1 perch (P.)

A *perch* of stone or masonry is $16\frac{1}{2}$ ft. (1 rd.) long, $1\frac{1}{2}$ ft. wide, and 1 ft. high.

80. Avoirdupois Weight.

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 1 short ton (T.)
2240 pounds	= 1 long ton (T.)

Avoirdupois weight is used in weighing all ordinary substances except precious metals, jewels, and drugs at retail.

The long ton is used in mining and in the United States custom-house.

81. Liquid Measure.

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)

A gallon contains 231 cu. in.

31.5 gal. are considered 1 barrel (bbl.).

63 gal. = 1 hogshead (hhd.).

82. Dry Measure.

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

A bushel contains 2150.42 cu. in.

A dry quart contains 67.2 cu. in.

A liquid quart contains 57.75 cu. in.

83. Measures of Time.

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
30 days	= 1 commercial month (mo.)
52 weeks	= 1 year (yr.)
12 months	= 1 year
360 days	= 1 commercial year
365 days	= 1 common year
366 days	= 1 leap year
100 years	= 1 century

84. Counting.

12 units	= 1 dozen (doz.)
12 dozen	= 1 gross (gro.)
12 gross	= 1 great gross (gr. gro.)
20 units	= 1 score

85. Wood Measure.

A straight pile of wood, 4 ft. × 4 ft. × 1 ft., contains 1 cord foot. 8 cord feet, or 128 cubic feet = 1 cord.

86. United States Money.

10 mills	= 1 cent
10 cents	= 1 dime
10 dimes	= 1 dollar
10 dollars	= 1 eagle

The mill is not a coin, but the term is frequently used in computations. The term "eagle" is not common in business.

87. English Money.

4 farthings (far.)	= 1 penny (*d.*), (plural, "pence")
12 pence	= 1 shilling (*s.*) = \$.243+.
20 shillings	= 1 pound sterling (£), \$4.8665

88. French Money.

100 centimes (c.) = 1 franc (fr.)

A franc is worth about $.193.

89. German Money.

100 pfennigs (pf.) = 1 mark (M.)

A mark is worth about $0.238.

Written Work

1. Draw a line 1 foot long at the blackboard. Try to form a very definite mental picture of this length so that you may be able to estimate lengths with a fair degree of accuracy.

2. Draw lines of the following lengths without any aid, then check each estimate.

$\frac{1}{2}$ ft., 2 ft., 3 ft., $\frac{2}{3}$ ft., $1\frac{1}{2}$ ft., 7 ft., 3 in., 9 in., 15 in.

3. Estimate the length and width of your desk; the length and width and height of the classroom; the dimensions of the school building and grounds. Check your estimates.

4. Practice estimating and stepping off various distances until you can do so with a fair degree of accuracy.

5. Lay off on the school grounds a square rod and, if possible, an acre. Estimate the areas of various lots, parks, or fields, and check your results, when possible.

6. By lifting various units of weights and then checking your own estimates of the weights of numerous objects, you can acquire the ability to estimate certain weights with but a small error.

90. Changing to Lower Denominations.

Example. How many quarts in 3 bu. 3 pk. 5 qt.?

SOLUTION.

3 bu. = 12 pk.
12 pk. + 3 pk. = 15 pk.
15 pk. = 120 qt.
120 qt. + 5 qt. = 125 qt.

Therefore, 3 bu. 3 pk. 5 qt. = 125 qt.

After studying this solution, state a method for reducing measured quantities to lower denominations.

Oral Work

Reduce as indicated ; do as much of the work orally as possible.

1. $15\frac{1}{3}$ yd. to feet.
2. $4\frac{2}{3}$ ft. to inches.
3. 5 gal. 3 qt. 1 pt. to pints.
4. 4 bu. 3 pk. 5 qt. to quarts.
5. 5 lb. 9 oz. to ounces (avoir.).

Example. Reduce .345 mi. to lower denominations.

SOLUTION.

1 mi. = 320 rd.
.345 mi. = 110.4 rd.

1 rd. = 5.5 yd.
.4 rd. = 2.2 yd.

1 yd. = 3 ft.
.2 yd. = .6 ft.

1 ft. = 12 in.
.6 ft. = 7.2 in.

Therefore, .345 mi. = 110 rd. 2 yd. 0 ft. 7.2 in.

After studying this solution, state a method for reducing fractions of a large denomination to units of lower denominations.

Written Work

Reduce :

1. $\frac{8}{9}$ mi. to rods.
2. .236 mi. to feet.
3. .89 bu. to quarts.
4. $\frac{13}{24}$ sq. mi. to square rods.
5. 62 rd. to feet.
6. .85 gal. to pints.
7. 4.3 mi. to feet.
8. A man bought an acre of land and sold it in building lots of 10 sq. rd. each. How many lots did he sell?

91. Changing to Higher Denominations. When a quantity is expressed in a low denomination, it may be desired to express it in higher denominations.

Examples. 1. Express 1316 pints in higher denominations.

SOLUTION. Since 2 pints = 1 quart,
1316 pints = 658 quarts.
Since 4 quarts = 1 gallon,
658 quarts = 164 gals., and 2 qt. remaining.
Therefore, 1316 pints = 164 gal., 2 qt.

After studying this solution, state a method for reducing quantities to units of higher denominations.

2. Reduce 3 qt. 1 pt. to a decimal part of a peck.

SOLUTION. 3 qt. 1 pt. = 7 pt.
1 pk. = 16 pt.
Therefore, 3 qt. 1 pt. = $\frac{7}{16}$ pk.
$\frac{7}{16}$ pk. = .4375 pk.

Written Work

Reduce to units of higher denominations:

1. 367 pints (dry measure).
2. 133 in.
3. 47 ft.
4. 12 yd.
5. 216 oz. (avoir.).
6. 31 pt. (liquid measure).
7. 2369 sq. in.
8. Reduce 4 sq. ft. 68 sq. in. to a decimal part of a square rod.
9. Reduce 1 qt. 2 pt. to a decimal part of a gallon.
10. Reduce 7 oz. to a decimal part of a pound (avoir.).
11. Reduce $\frac{2}{3}$ yd. to a decimal part of a rod.
12. Reduce $2\frac{1}{2}$ ft. to a decimal part of a yard.
13. Reduce 47 sq. rd. to a decimal part of an acre.
14. Reduce $8\frac{1}{2}$ in. to a decimal part of a foot.
15. Reduce $2\frac{1}{2}$ qt. to a decimal part of a gallon.
16. Reduce 4 ft. 7 in. to a decimal part of a rod.

92. Addition of Denominate Numbers.

Example. Find the sum of 2 yd. 2 ft. 9 in. and 4 yd. 1 ft. 7 in.

SOLUTION.

	2 yd.	2 ft.	9 in.
	4 "	1 "	7 "
	6 yd.	3 ft.	16 in.
or	7 "	1 "	4 "

State a method for adding denominate numbers.

Written Work

Add:

1. 2 yr. 5 mo. 18 da.
   5 " 8 " 19 "
   6 " 11 " 13 "

2. 5 gal. 3 qt. 1 pt.
   4 " 2 " 1 "
   5 " 1 " 1 "

3. 2 lb. 5 oz. 17 pwt.
   3 " 8 " 19 "
   4 " 9 " 12 "

4. 5 A. 126 sq. rd.
   16 " 249 " "
   13 " 168 " "

5. 7 mo. 12 da.
   5 " 5 "

6. 12 mi. 4 rd.
   8 " 9 "

7. A rectangular field is 18 rd. 2 yd. 2 ft. long, and 13 rd. 1 yd. $2\frac{1}{2}$ ft. wide. What length of wire will be required for a fence 6 wires high?

8. A dealer bought 17 gal. 3 qt. of milk from each of two men, and 29 gal. 3 qt. 1 pt. from a third man. What was the entire cost at 19 cents a gallon?

93. Subtraction of Denominate Numbers.

If it is required to subtract 1 mi. 42 rd. 13 ft. from 3 mi. 25 rd. 14 ft., we write the numbers as follows:

3 mi. 25 rd. 14 ft.
1 " 42 " 13 "

When, as in this problem, the number of units of some denomination of the minuend is smaller than the number of corresponding units of the subtrahend, combine one unit of the next larger denomination with the number of units of the minuend. The problem thus becomes:

2 mi. 345 rd. 14 ft.
1 " 42 " 13 "
1 " 303 " 1 "

Written Work

Subtract as indicated:

1. 13 bu. 3 pk. 2 qt.
   6 " 2 " 7 "

2. 7 A. 5 sq. rd. 6 sq. ft.
   3 " 7 " " 2 " "

3. 27 rd. 4 yd. 1 ft.
   13 " 5 " 2 "

4. 14 gal. 2 qt. 1 pt.
   5 " 3 "

5. 19 lb. 12 oz.
   7 " 14 "

6. A man owned a field containing $1\frac{2}{5}$ A. From it he sold two lots each having an area of $11\frac{1}{2}$ sq. rd. What was the area of the part remaining?

7. From a cask which contained 31 gal. 3 qt. 1 pt. of vinegar, 19 gal. 3 pt. were drawn out. How much remained?

94. Multiplication Involving Denominate Numbers.

Example. Multiply 3 yd. 2 ft. 7 in. by 4.

SOLUTION.

	3 yd.	2 ft.	7 in.
			4
	12 yd.	8 ft.	28 in.
or	15 yd.	1 ft.	4 in.

Written Work

Multiply as indicated:

1. 5 gal. 3 qt. 1 pt. by 7.
2. 7 lb. 5 oz. by 12.
3. 5 cu. yd. 19 cu. ft. 364 cu. in. by 13.
4. 4 mi. 19 rd. 3 yd. by 18.

95. Division Involving Denominate Numbers.

Examples. 1. Divide 4568 inches by 8.

SOLUTION. 4568 in. ÷ 8 = 571 in.
571 in. = 2 rd. 8 yd. 2 ft. 7 in.

2. Divide 17 gal. 2 qt. 1 pt. by 3.

SOLUTION. 17 gal. 2 qt. 1 pt. = 141 pt.
141 pt. ÷ 3 = 47 pt.
47 pt. = 5 gal. 3 qt. 1 pt.

3. How many times is 4 gal. 2 qt. contained in 1 barrel?

SOLUTION. 4 gal. 2 qt. = 18 qt.
1 bbl. = 126 qt.
126 qt. contains 18 qt. 7 times.

Explain the method used in this illustration.

Written Work

Divide as indicated:

1. 27 cu. yd. 15 cu. ft. ÷ 6.

2. 584 A. 8 sq. rd. ÷ 8.

3. 7 sq. ft. 48 sq. in. ÷ 16.

4. How many times are 4 yd. 2 ft. 7 in. contained in 29 yd. 6 in.?

5. Reduce to the decimal part of a mile: 5 rd.; $7\frac{1}{2}$ rd.; 5 ft.; 13 yd.; 5 ft. 9 in.

6. An automobile traveled 1 mi. in $3\frac{1}{2}$ min. What was the rate per hour?

7. If a man sells 17 loads of wheat, each containing 53 bu. 3 pk., at $97\frac{1}{2}$ cents a bushel, how much should he receive?

8. Fifteen cans hold an average of 10 gal. 2 qt. 1 pt. How much do they all hold?

9. Change .427 sq. mi. to units of lower denominations.

10. A cubic foot of water weighs 62.5 lb., cast iron weighs about 7.2 as much as water; what is the weight of $2\frac{3}{8}$ cu. ft. of cast iron?

11. Determine whether the window space in your school room is as much as $\frac{1}{6}$ of the floor space. It should be at least $\frac{1}{6}$ in a well-lighted room.

12. A train leaves a city at 4 : 30 P.M., and reaches a certain station $87\frac{1}{2}$ mi. distant at 6 : 20 P.M. Allowing 12 min. for station stops, what is the rate of travel per hour?

13. There are twenty-four students in a class and each needs $2\frac{1}{2}$ cups of hot water for a cooking lesson. How much water must there be in a kettle to supply all the class at one time, allowing 4 cups to a quart?

CHAPTER XI

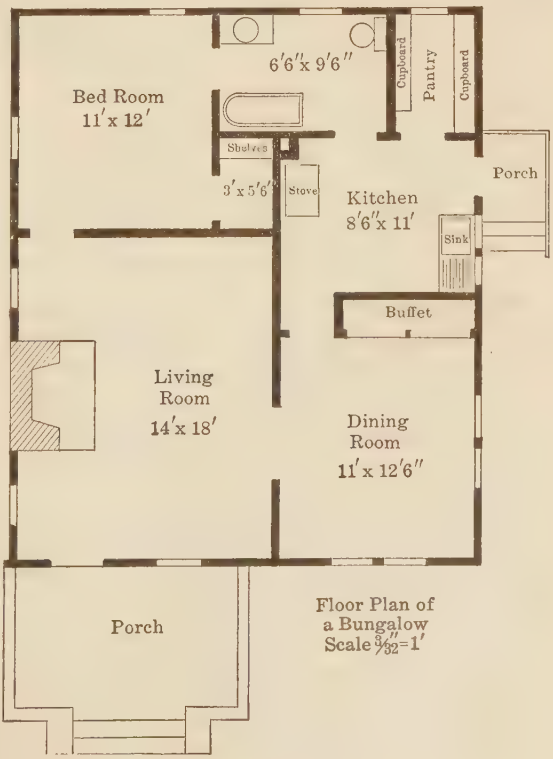
DRAWINGS AND GRAPHS

NEWSPAPERS, magazines, and trade journals make frequent use of drawings or graphs in order to make clear the relations between magnitudes. A graph will often reveal a relationship much more quickly and more clearly than a table of statistics.

96. Purposes of Drawings and Graphs. Drawings and graphic representations are extensively employed for two purposes:

a. For reference, to show quickly and conveniently the shape and dimensions of fields, rooms, buildings, furniture, machinery, etc.

Dimensional drawings and diagrams, similar to this one, are of great value. An architect designs a building, and the contractor builds it in accordance with the plan shown by the drawing. Tools and machinery are usually constructed in accordance with drawings prepared by draftsmen and designers.

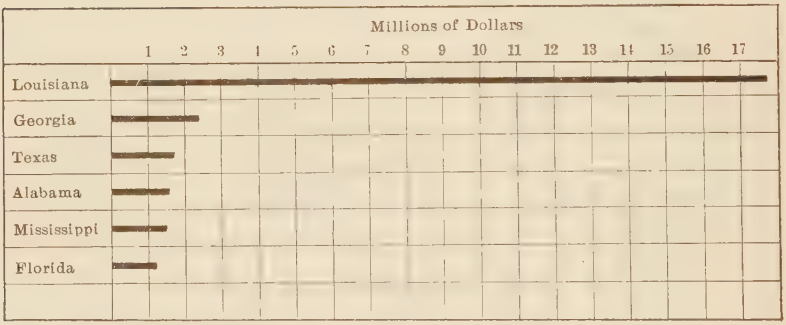


b. To present statistics in a manner

which will show their meaning and their relationship more clearly than can be done by columns of figures. For example, the relative importance of Louisiana as a producer of cane sugar is made clearer by the following graph than by the table of statistics.

VALUE OF CANE SUGAR PRODUCED IN CERTAIN STATES IN 1909

STATE	DOLLARS
Louisiana	17,752,537
Georgia	2,268,110
Texas	1,669,683
Alabama	1,527,166
Mississippi	1,506,887
Florida	1,089,698



97. Drawing to Scale. The ability to read drawings and graphic representations intelligently depends somewhat upon a knowledge of how to draw figures to scale.

The values represented by the lines in drawings and graphs can be easily determined because the lines are drawn to scale ; that is, a certain distance on the drawing represents a specified amount of the quantity represented.

In the floor plan, on page 103, one foot of true distance is represented by $\frac{3}{32}$ of an inch plotted distance on the drawing. All lines in the drawing correspond to this scale.

In the graph showing the production of cane sugar, $\frac{3}{8}$ inch represents a value of $1,000,000.

98. Types of Scales. The scale may be stated in various ways, the most common of which are :

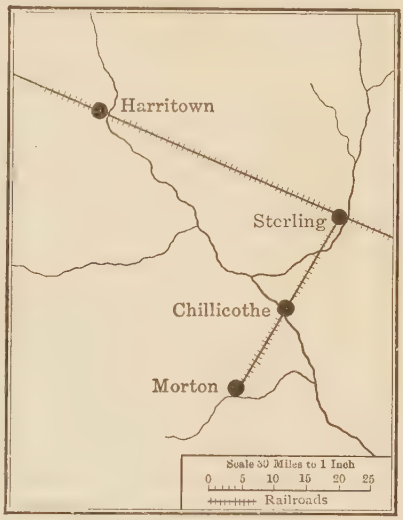
a. A fraction.

For example, a scale of $\frac{1}{4}$ means that the true distances or values are four times as great as the plotted distances or values.

b. Statement of equivalent dimensions or values.

For example, $1'' = 1'$. (Read 1 in. = 1 ft.) One inch of the drawing represents one foot of the real object. In the graph showing cane sugar production the scale is $\frac{3}{8}'' =$ \$1,000,000, that is, $\frac{3}{8}$ inch represents \$1,000,000.

c. A diagram or scaled rule accompanying the drawing. This is the method commonly used in map drawing. Thus, in the map shown here, the scale is indicated.



d. The graph may be drawn on ruled paper, the distance between the ruled lines representing a specified value.

Exercise

1. A scale of $1'' = 1'$, is equivalent to what fractional scale?
2. A scale of $1'' = 5''$, is equivalent to what fractional scale?
3. A scale of $1'' = 5'$, is equivalent to what fractional scale?
4. If the scale is $\frac{1}{12}$, what distance on the drawing would represent 6′ 3″? 4′ 9″?
5. If the scale is $\frac{1}{300}$, what plotted distance would represent 750 feet? 750 inches?
6. If the scale is $1'' = 1000$ bushels, what distance on a graph would represent 5125 bushels? 8500 bushels? 1750 bushels?

The following floor plan is drawn to a scale of $\frac{1}{96}$.

7. One inch of plotted distance on the floor plan is equivalent to how many feet of true distance?

8. One foot of true distance on the floor plan is represented by what part of an inch on the drawing?

9. What are the true dimensions of this room?

10. Assume that this drawing is made to the scale, $\frac{1}{8}$ in. = 1 ft. ($\frac{1}{8}'' = 1'$). What are the true dimensions of the room?

11. The map on p. 105 is drawn to the scale $1'' = ?$ miles.

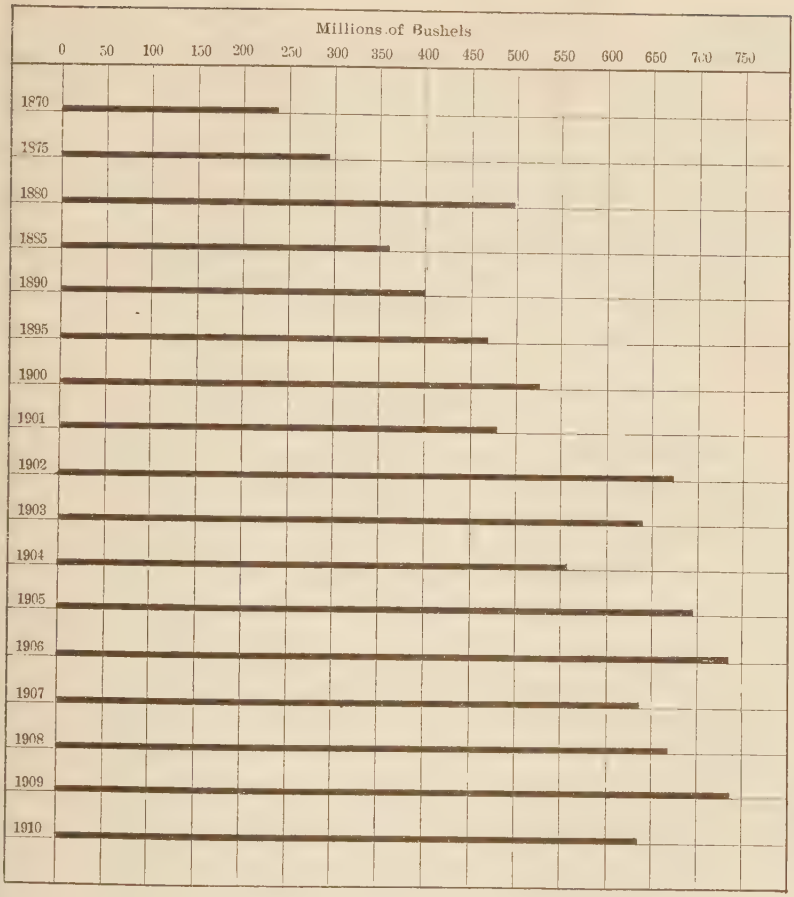
12. $\frac{1}{8}''$ on this map represents what true distance?

13. What is the true distance between each of the towns on the railroad? Measure distances from centers of towns.

14. An electric railroad is in process of construction between Morton and Harritown. It is surveyed in a direct line. How much shorter will this road be than the one via Sterling?

15. It is planned to locate a power house halfway between Sterling and Harritown. Indicate its position on the map and determine its distance from the two towns. How far will it be from the nearest point on the new electric line?

The following is an illustration of a graph drawn on ruled paper. The figures along the top of the graph indicate the number of millions of bushels of wheat grown. Quantities smaller than 10,000,000 bushels can be approximated with sufficient accuracy for general comparative purposes. For example, the wheat crop of 1870 is shown by the graph to have been about 236,000,000 bushels, and that of 1904, about 553,000,000 bushels.



Oral Work

1. What was the approximate crop of each of the years shown?
2. What was the record crop shown by the graph?

99. Determining the Scale. When a drawing or a graph is to be made, it is first necessary to determine the scale. The scale depends, in general, upon three things:

a. The size of the paper to be used.

b. The largest dimension to be shown on the graph.

c. Convenience in showing fractional parts of the scale.

Example. A room is 15′ 6″ long by 10′ 3″ wide. What scale should be used to show a diagram of this room on a sheet of paper 8″ by 10″, leaving a margin of at least one inch around the drawing?

SOLUTION. The largest dimension of the room is 15′ 6″, or 186″.

The largest dimension of the paper is 10 inches. After deducting 2 inches for margins, there is a space of 8″ available for the drawing.

The largest scale possible, therefore, is 8″ represents 186″.

This is equivalent to a scale of 1″ = 23¼″. (How was this determined?)

For convenience in showing fractional parts of a foot, it will be better to use the scale of 1″ = 24″.

Written Work

1. With the scale 1″ = 2′, what length lines would represent the dimensions of the room 15′ 6″ × 10′ 3″?

2. Use this scale to make a drawing of a room 12′ 6″ by 9′ 9″.

3. A field is 120 rods by 80 rods. Scale, 1″ = 1 rod. How large a paper is needed to diagram the field, leaving a margin of 2 inches all around? Would this be a reasonable scale to use?

4. If a scale 1 inch = 20 rods is used, what size paper is necessary in order to diagram this field and leave a margin of one inch on all sides?

5. What is the largest scale that could be used to plot this field on a paper the size of this page, leaving margins of at least ¾″?

6. It is desired to show by a graph the imports of coffee into various countries of the world. The year chosen is 1910. The United States was the largest importer with an importation in round numbers of 804,000,000 pounds. The graph is to be placed on paper the same size as this page, with the same margins. How long a line would represent 100,000,000 bushels?

7. If the names of the countries require one inch, the scale might be —— inch = 100,000,000 bushels.

8. Draw a diagram of your school room, entering the dimensions as illustrated in the drawing on page 103.

9. Draw a diagram of a room in your home. Do not enter the dimensions but state the fractional scale used, and draw all lines very carefully in accordance with this scale.

NOTE TO TEACHER. When these drawings are brought to class, they may be exchanged, and the students required to compute the dimensions in accordance with the scale stated.

10. Draw a line graph (similar to the one on page 194) to express the following statistics.

EXPORTS OF TEA FROM VARIOUS COUNTRIES IN 1910

COUNTRY	POUNDS
British India	258,000,000
Ceylon	182,000,000
China	207,000,000
Dutch East Indies	33,000,000
Formosa	24,000,000
Japan	40,000,000
Singapore	2,000,000

Rule the paper thus:

Use the scale $\frac{1}{4}'' = 10$ million pounds. (How large a paper will you need?)

Arrange the countries so that the longest line will be at the top and the shortest at the bottom, the others being placed in the order of their lengths.

VARIOUS TYPES OF GRAPHS

100. Colored and Shaded Graphs. By the use of different colors of ink, or different shadings, comparisons may be made to show increase, decrease, or various changes in statistics from year to year.

The following graph shows the average number of wage earners employed in manufacturing industries in 1899 and 1909. The graph is limited to the ten states employing the largest number of men.

AVERAGE NUMBER OF WAGE EARNERS, BY STATES: 1909 AND 1899[1]

Oral Work

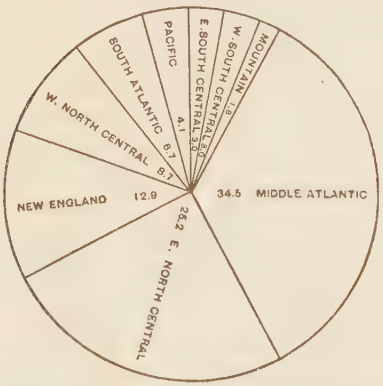
1. By referring to the graph, determine the approximate number of wage earners in each state in 1899 and in 1909.

2. Which state shows the greatest increase in the number of wage earners between the years 1899 and 1909?

101. The Circle is frequently used for graphic purposes. It is particularly valuable because it shows two things very clearly:

a. The relation of each magnitude to each of the others.

b. The relation of each magnitude to the sum of all.



VALUE OF MANUFACTURED PRODUCTS[1]

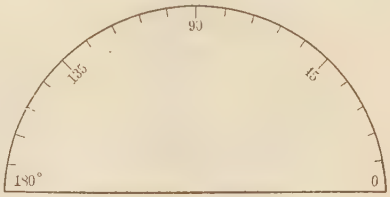
This graph shows clearly that the states of the Middle Atlantic Division produce a greater value of manufactured articles than the states of any other division, and that they produce about one third of the entire value of the manufactures of the country.

The scale used in making circular graphs is based on the degree. If the circumference of

[1] Reprinted by permission of the United States Bureau of the Census, Department of Commerce.

a circle were divided into 360 equal parts, each part would be one degree of arc. Degrees may be measured by an instrument called a protractor.

The method of locating the lines in a circle graph may be explained by showing how the space occupied by the Middle Atlantic section was determined.



This section produces .345 of the total value of manufactured products. It should, therefore, occupy .345 of the circle.

.345 of 360° is 124.2°.

Points as nearly as possible 124.2° apart are marked off on the circumference with the aid of the protractor, and lines are drawn from the center of the circle to these points on the circumference.

Written Work

1. The total exports of coffee from all the countries of the world in 1910 were 2,163,764,874 pounds. Brazil exported 1,286,217,168 pounds or —— of the world's trade. It should therefore occupy what part of a circle designed to show the coffee exports of the world? How many degrees of arc?

2. The next largest exporter was the Netherlands, with 173,823,451 pounds. What part of the world's supply was provided by this country? What part of the circle should represent the amount of coffee exports of the Netherlands?

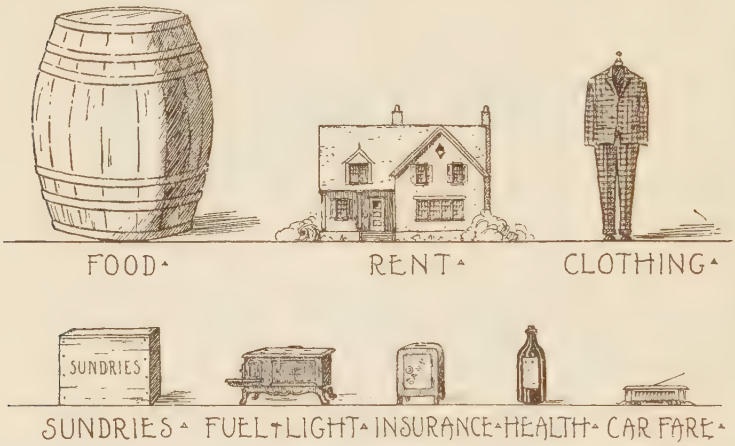
3. Five departments of a wholesale store produced profits as follows:

Dept.	Profits
I	$2165.00
II	3297.00
III	794.00
IV	1719.00
V	4625.00

Prepare a circle graph showing what per cent of the entire profit was made by each department.

102. Graphic Pictures. Another kind of graph frequently used is illustrated below:

WORKINGMEN'S EXPENDITURES



Food	$335.82
Rent	161.36
Clothing	98.79
Sundries	60.28
Fuel and light	36.94
Insurance	18.24
Health	14.02
Car fares	10.53

From *The Independent.*

These figures are based on a study of the wages of a number of workingmen, whose average annual income was $749.83 and whose average annual expenditure was $735.98.

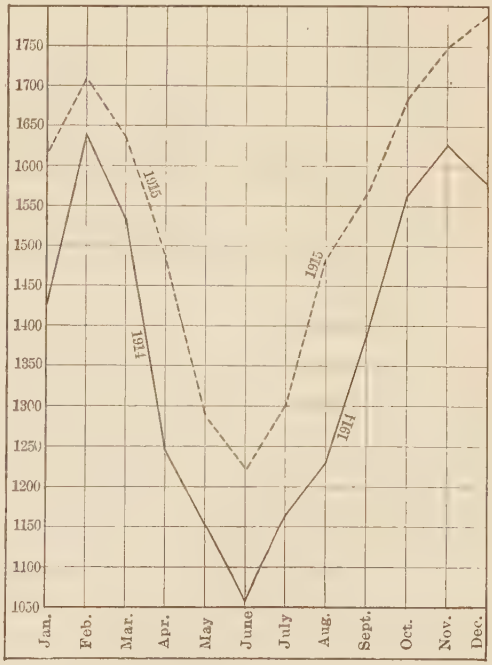
103. The Graphic Curve. Graphs drawn on cross-sectioned paper are especially valuable when it is desired to show the variation of statistics during consecutive intervals, as from month to month, or from year to year. The divisions on the vertical line may represent multiples of some unit of value and those on the horizontal line represent various intervals of time.

The following graph shows the variations in the monthly sales of a store during two years. The unbroken line ————————

indicates the sales for the year 1914 and represents the following statistics:

Sales for January	$1426.80	Sales for July	$1167.85
February	1638.80	August	1232.65
March	1526.75	September	1386.64
April	1246.70	October	1566.45
May	1147.10	November	1626.30
June	1060.50	December	1575.25

It is sufficient for comparative purposes to show the approximate sales in dollars.



104. How this Graph is Drawn. An unbroken line represents the data for 1914. A point was placed on the January line at a point representing approximately $1425.00. A second point was placed on the February line at a point representing approximately $1638.00. Other points were properly located on the other lines, and an unbroken line, or curve, was then drawn connecting all the points. In the same way the dotted line was drawn to represent the data for 1915.

Exercise

Study the graph and answer the following questions:

1. Was there any uniformity of trade from year to year? That is, did the busy and the dull months follow in about the same order each year?

2. As a rule, during what seasons did this store have the largest trade?

3. What was its "dullest" month?

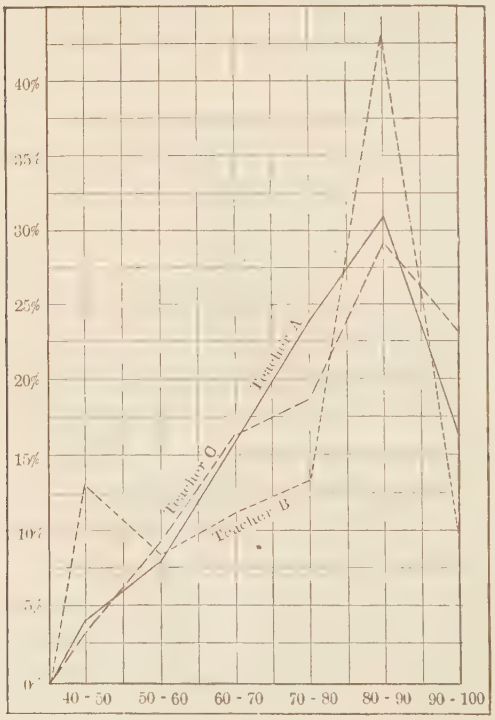
4. How did the business of 1915 compare with that of 1914?

5. About how many more dollars' worth of goods were sold during August, 1915, than during August, 1914?

6. Estimate the sales for each month of the year 1914. If your answers are within $2.00 of the correct amount, they will be accurate enough for purposes of comparison.

7. Discuss the value of such a graph.

105. The following graph shows a slightly different use of the curve. The curves serve to compare the grades given by three teachers in a certain school, showing what per cent of the students in their classes received certain grades.



This graph was made in the following manner:

A table was prepared showing what per cent of each teacher's students received grades between 40 and 50, 50 and 60, etc.

For example: Eight students of teacher A received grades 60–70. This was 16% of the students in A's classes.

Grade	Teacher A		Grade	Teacher A	
	No. of Students	Per Cent of Total Number		No. of Students	Per Cent of Total Number
40–50	2	4	70–80	12	24
50–60	4	8	80–90	16	32
60–70	8	16	90–100	8	16

The curve for Teacher A was plotted thus:

On the horizontal line above 40–50, a point was placed to indicate 4%.

On the horizontal line above 50–60, a point was placed to indicate 8%, etc.

All of the points thus located to represent grades given by A were then connected.

Oral Work

Interpret the curves, stating what per cent of the students of each teacher received the grades specified.

Written Review

Use paper $8\frac{1}{2}''$ by $11''$, or larger, for the following graphs.

1. By a graph similar to the one on page 110 show the following statistics of exports from the United States to the United Kingdom. Use black ink for the data of 1910; red ink for those of 1911.

	1910	1911
Cattle	£ 2,578,000	£ 3,056,000
Bacon	4,453,000	5,067,000
Hams	2,329,000	2,712,000
Fresh Beef	1,070,000	397,000
Lard	4,201,000	4,014,000
Leather	4,057,000	3,828,000
Machinery	2,287,000	2,894,000
Copper	2,568,000	3,027,000
Paraffin Wax	871,000	617,000
Petroleum	3,745,000	3,370,000
Tobacco	2,815,000	3,278,000
Fish	1,021,000	702,000

2. Show graphically what part of the coinage of the years named in the following table was gold, what part silver, and what part other coin.

	Gold	Silver	Other Coin
1909	88,776,908	8,087,852	1,756,389
1910	104,723,735	3,740,468	3,036,929
1911	56,176,822	6,457,301	3,156,726
1912	17,498,522	7,340,995	2,577,386

3. By drawings of corn stalks, ears of corn, corn cribs, or any other suitable design, graph the following statistics of corn crops in the United States.

Year	Bushels	Year	Bushels
1870	1,094,000,000	1900	2,105,000,000
1880	1,717,000,000	1910	2,886,000,000
1890	1,489,000,000		

4. By a graph similar to the illustration of monthly sales (p. 113), show the variation in the wholesale price of corn per bushel by months for the following years.

	1910	1911	1912
	Cents	Cents	Cents
January	62.3	48.2	62.2
February	65.2	49.0	64.6
March	65.9	48.9	66.6
April	65.5	49.7	71.1
May	63.5	51.8	79.4
June	65.2	55.1	82.5
July	66.2	60.0	81.1
August	67.2	65.8	79.3
September	66.3	65.9	77.6
October	61.1	65.7	70.2
November	52.6	64.7	58.4
December	48.8	61.8	48.7

5. Show by a graph the data for the attendance at your school for a school year. (The principal can supply the data.)

6. Graph the temperatures for a certain hour of several successive days.

7. Make a graph showing your progress in speed and accuracy in addition, subtraction, multiplication, and division for a given set of examples.

8. What is the largest scale that could be used to represent a square farm of 160 acres on a paper 18″ by 20″, leaving a margin of at least 1″?

9. Two cities known to be 60 miles apart are $2\frac{1}{2}$ inches apart on a map. What is the scale of the map?

CHAPTER XII

PERCENTAGE

106. Relation of Percentage to Common and Decimal Fractions. **Percentage** is the process of computing by hundredths. The symbol % stands for per cent.

Thus, $15\,\%$ means $\frac{15}{100}$ or .15.

$8\,\%$ means $\frac{8}{100}$ or .08.

$300\,\%$ means $\frac{300}{100}$ or 3.00

$\frac{1}{2}\,\%$ means $\frac{\frac{1}{2}}{100}$ or $.00\frac{1}{2}$ or .005.

$\frac{2}{3}\,\%$ means $\frac{\frac{2}{3}}{100}$ or $.00\frac{2}{3}$.

$7\frac{1}{2}\,\%$ means $\frac{7\frac{1}{2}}{100}$ or $.07\frac{1}{2}$ or .075.

Oral Work

Express each of the following as a common fraction and as a decimal fraction:

1. $7\,\%$	**2.** $4\,\%$	**3.** $19\,\%$	**4.** $24\,\%$
5. $2\frac{1}{3}\,\%$	**6.** $\frac{1}{4}\,\%$	**7.** $\frac{1}{10}\,\%$	**8.** $200\,\%$
9. $184\,\%$	**10.** $5\frac{1}{6}\,\%$	**11.** $\frac{1}{100}\,\%$	**12.** $16\frac{2}{3}\,\%$

Express with the symbol %:

13. .74	**14.** .03	**15.** $.07\frac{1}{4}$	**16.** 2.00
17. $.32\frac{1}{5}$	**18.** .001	**19.** 1.00	**20.** .0045

21. .5	**22.** .005	**23.** $.16\frac{2}{3}$	**24.** $.14\frac{2}{7}$
25. $\frac{1}{2}$	**26.** $\frac{1}{4}$	**27.** $\frac{1}{5}$	**28.** $\frac{3}{2}$
29. $\frac{3}{4}$	**30.** $\frac{2}{5}$	**31.** $\frac{4}{5}$	**32.** $\frac{1}{3}$

33. 40 % of a quantity is how many hundredths of it? What fractional part of it?

34. 100 % of a number is how many times the number?

35. 300 % of a number is how many times the number?

36. 140 % of a number is how many times the number?

37. 2 times a number is what per cent of the number?

38. 5 times a number is what per cent of the number?

39. What part of a number is:

20 % of it?	12 % of it?	13 % of it?
80 % of it?	75 % of it?	$\frac{1}{5}$ % of it?
100 % of it?	$\frac{1}{2}$ % of it?	$12\frac{1}{2}$ % of it?
$37\frac{1}{2}$ % of it?	$87\frac{1}{2}$ % of it?	50 % of it?
$6\frac{2}{3}$ % of it?	$62\frac{1}{2}$ % of it?	$66\frac{2}{3}$ % of it?

40. Express each of the following as a decimal fraction and with the symbol %.

$\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{2}{5}$, $\frac{1}{4}$, $\frac{3}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{1}{10}$, $\frac{5}{12}$, $\frac{7}{8}$, $\frac{3}{4}$.

41. The following table states the fractional part of the total number of colonies of bees in the United States owned in each section of the country. State what per cent of the colonies are owned in each section.

New England	.012
Middle Atlantic	.085
East North Central	.158
West North Central	.159
South Atlantic	.197
East South Central	.147
West South Central	.11
Mountain	.05
Pacific	.082

42. The following table shows the per cent of the total value of live stock owned in each section of the United States. State these per cents as decimal fractions.

New England	2 %
Middle Atlantic	7.1 %
East North Central	19.8 %
West North Central	31.5 %
South Atlantic	7.4 %
East South Central	7.5 %
West South Central	12 %
Mountain	7.9 %
Pacific	4.8 %

107. Terms Used in Percentage.

Base. The number of which a given per cent is to be taken is called the **base.**

Rate. The per cent of the base to be taken is called the **rate.**

Percentage. The result obtained by taking a certain per cent of the base is called the **percentage.** Thus,

$$64 \times .25 = 16;$$

or **base × rate = percentage.**

Fundamental Processes of Percentage

Computations in percentage are based on the following:

$$\text{base} \times \text{rate} = \text{percentage.}$$
$$64 \times .25 = 16.$$

Since the percentage is the product of the base and rate, it is evident that,

a. The percentage divided by the base equals the rate. For example: $16 \div 64 = .25 = 25\ \%$.

b. The percentage divided by the rate equals the base. For example: $16 \div 25\ \% = 16 \div .25 = 64$.

From these statements it is evident that if any two of the terms are known, the other one can be found.

In the chapter on Decimal Fractions you learned how to multiply and divide by any number expressed decimally. Percentage

involves the mathematical principles of decimal fractions. The exercises in decimal fractions involved computations with tenths, hundredths, thousandths, etc. In Percentage, computation is by hundredths.

All mathematical computations involved in percentage may be grouped under three headings:

a. To find a given per cent of a number.

b. To find what per cent one number is of another.

c. To find a number when a certain per cent of it is known.

108. To find a given per cent of a number; that is, to find the percentage.

Examples. 1. Find 1 % of 428.

SOLUTION. 1 % = .01. .01 of 428 = 4.28.

2. Find 7 % of 378.

SOLUTION. .07 × 378 = 26.46.

3. Find 14 % of $360.

SOLUTION.

$360
.14
1440
360
$50.40

To find the percentage, multiply the base by the rate.

Oral Work

1. Find 1 % of each of the following: 18; 146; 198; 7; 03; 876; .045.

2. Find 5 % of each of the following: 80; 60; 25; 42.

3. Find 10 % of $18.20; of 478 mi.; of 32 yd.; of 648 A.

4. How many places should be pointed off in finding each of the following:

17 % of 4762?	17 % of 476.2?	17 % of 47.62?
1.7 % of 4762?	.17 % of 4762?	17 % of .4762?

5. Find 10 % of 400; 560; 8.46.

6. Find 3 % of 40; 60; 15.
7. Find 100 % of 15; 40; 3.
8. Find 20 % of 80; 70; 60.
9. Find 50 % of 40; 2; 10.
10. Find 30 % of 30; 40; 80.
11. Find $12\frac{1}{2}$ % of 480; 96; 640.
12. Find $37\frac{1}{2}$ % of 24; 80; 72.
13. Find 200 % of 9; 32; $18\frac{1}{2}$.
14. Find $14\frac{2}{7}$ % of 21; 35; 350.
15. Find 25 % of 12; 17; 96.
16. Find $\frac{1}{2}$ % of 120; 34; 18.

Written Work

Find:

1. 17 % of 472.
2. 92 % of $537.
3. $16\frac{2}{3}$ % of $420.
4. 3 % of 48,729.
5. 200 % of 37 ft.
6. 1.8 % of 42,690.
7. 17.5 % of 479,362.
8. 250 % of $142.
9. 128 % of 346 yd.
10. 5 % of 83.6.
11. $7\frac{1}{2}$ % of 42.736.
12. $\frac{1}{2}$ % of 8476.
13. 32 % of $4.76.
14. 19 % of .084.
15. 25 % of 264 sq. mi.
16. $37\frac{1}{2}$ % of 24 A.
17. $12\frac{1}{2}$ % of $488.
18. $87\frac{1}{2}$ % of 648 mi.
19. $33\frac{1}{3}$ % of $\frac{1}{5}$.
20. 20 % of $\frac{7}{8}$.
21. $6\frac{2}{3}$ % of 32 cu. ft.
22. 40 % of 85 sq. rd.
23. $62\frac{1}{2}$ % of 40 A.
24. $33\frac{1}{3}$ % of $36.
25. $66\frac{2}{3}$ % of 120 ft.
26. $12\frac{1}{2}$ % of 64 yd.

27. A school has an enrollment of 450 students. How many students are absent when 4 % are absent?

28. What is $200 increased by 4 % of itself?

29. In 1900 the population of a certain city was 52,840. In 1910 the population was 15 % larger than in 1900. What was the population in 1910?

30. A boy made a journey of 48 miles. He rode 85 % of the distance in an automobile and walked the remaining distance. How far did he walk?

31. An architect's fees for designing a house are 4 % of its cost. The cost is $7360. What is the fee?

32. A man whose salary is $3200 spends 19 % of it for rent. How much does he spend for rent?

33. A clerk receives 2 % of the amount of his sales. If his sales for Monday were $160.40, how much did he receive?

34. The report of a school with an attendance of 400 showed the per cent of tardiness to be $1\frac{1}{2}$. How many were tardy?

35. A man earns $1400 a year and saves 18 % of it. How long will it take him to save $756?

36. In a recent year, the following live stock was reported on farms in the United States:

	Number
Cattle	61,803,866
Horses	18,833,113

The following table shows the per cents of the total number of cattle and horses, in terms of the above data, in each section of the country. Thus 5.2 % of 61,803,866 was the number of cattle in the Pacific States.

Section	Cattle	Horses
New England	2.2 %	1.8%
Middle Atlantic	6.8	6.2
East North Central	15.9	22.2
West North Central	28.6	34.3
South Atlantic	7.8	5.6
East South Central	6.4	5.8
West South Central	17.3	11.8
Mountain	9.8	7.2
Pacific	5.2	5.1

Rule a form with the headings suggested below. Enter the facts given above; find the number of cattle and horses in each section of the country and enter the statistics on the blanks.

Cattle and Horses in United States

Section	Cattle		Horses	
	United States Total 61,803,866		United States Total 18,833,113	
	Per Cent	Number	Per Cent	Number

109. To find what per cent one number is of another; that is, to find the rate.

Method. Since the percentage is the product of the base and the rate, and since the product may be divided by one factor to determine the other factor, we have the following:

percentage ÷ base = rate.

Examples. 1. 6 is what per cent of 12?

SOLUTION. $\frac{6}{12} = \frac{1}{2} = .50$ or 50 %.

2. A farmer had 72 sheep and sold 18 of them. What per cent of his flock did he sell?

SOLUTION. $18 \div 72 = .25$ or 25 %.

3. George had one brother and three sisters. What per cent of the children were boys?

SOLUTION. There were 5 children in the family.
$\frac{2}{5}$ of the children were boys.
$2 \div 5 = .4$ or 40 %.

Oral Work

1. What per cent of 8 is 4?
2. What per cent of 8 is 2?
3. What per cent of 24 is 3?
4. What per cent of 12 is 18?
5. What per cent of 9 is 3?
6. What per cent of 25 is 5?
7. What per cent of 25 is 20?
8. What per cent of 15 is 30?
9. What per cent of 10 is 8?
10. What per cent of 50 is 20?
11. What per cent of 60 is 40?
12. 15 is what per cent of 20?
13. 20 is what per cent of 40?
14. 18 is what per cent of 24?
15. 1 ft. is what per cent of 1 yd.?
16. 20 bu. are what per cent of 60 bu.?

17. 4 mo. are what per cent of 1 yr.?

18. 3 da. are what per cent of 1 wk.?

19. If you sleep 8 hr. out of 24, what per cent of the time do you sleep?

20. If a man earns $24 a week and saves $4, what per cent of his earnings does he save?

21. An agent sold goods amounting to $1800 and received $200 for his services. What per cent did he receive on the sales?

Written Work

1. A baseball team played 60 games and won 42. What per cent of the games played did the team win?

2. In a school of 250 students 45 were absent. What per cent of the students was absent?

3. In a school auditorium there are 650 seats. Four hundred eighty of the seats are reserved for the students. What per cent of the seats is not reserved?

4. A building worth $14,500 is insured for $10,150. For what per cent of its value is it insured?

5. In a spelling lesson a boy spelled all but 4 of the 30 words correctly. What per cent of the words did he spell correctly?

6. A corporation has a capital stock of $164,000. Dunham owns $41,000 of the stock. What per cent of the stock does he own?

7. A business man invested $3000. His income from the investment the first year was $420. What per cent did he earn?

8. In a certain school there are 224 boys and 312 girls. What per cent of the total enrollment is boys? girls?

9. Eighteen of the boys in this school failed to pass in at least one study. What per cent of the boys failed?

10. Twenty-three of the girls failed. What per cent of the girls failed?

11. What per cent of the entire school failed?

12. In 1910, the total population of the United States, 10 years of age and over, was 71,580,270. Of this number, 5,517,608 were unable to either read or write. What per cent of the population was illiterate?

13. The total population of the United States in 1910 was 91,972,266. The number of foreign-born inhabitants was 13,345,545. What per cent of the total population was foreign born?

14. The largest number of foreign-born people were Germans. They numbered 2,501,181. What per cent of the entire population was of German birth?

15. What per cent of the foreign-born population was of German birth?

16. The forests of the United States originally contained about 5200 billion board feet of timber. Cutting, clearing, and fire have reduced the stand to about 2500 billion board feet. What per cent of the timber remains?

17. The following table is similar to one used by a clothing store to determine the tailoring firm which furnishes suits having the best sale. Record is kept of the number of suits purchased from each firm, and of the number of these suits sold.

Find what per cent of the suits purchased from each of the manufacturers was sold, and enter all statistics on a ruled form similar to the model.

Of what value is this table of statistics to the merchant?

PER CENT OF SALES FROM STOCK PURCHASED FROM CLOTHING MANUFACTURERS

MANUFACTURER	NUMBER	NUMBER SOLD	PER CENT SOLD
TAYLOR, BRIGHT & CO	368	293	
WELLS & BAILEY	493	316	
F. H. BAINUM	639	582	
DUDLEY & COLE	523	416	
WILLIAMS, ELDRIDGE & CO.	863	719	
M. J. HARTMAN & SONS	1392	1202	
W. C. DENNISTON & CO.	1020	905	

18. The following table shows one of the ways in which merchants determine which clerk is selling the most goods.

Rule a form similar to the model; enter on the blank the facts which are given in the model; find the total sales made by all of the salesmen. Find what per cent of the total sales was made by each salesman. Enter all of these facts on the blank.

TABLE SHOWING PER CENT OF MONTH'S SALES MADE BY VARIOUS SALESMEN

SALESMAN'S NAME	SALES	PER CENT OF TOTAL
J B. DENNY	$ 1264 90	
O.F. ALBRIGHT	1913 52	
G.F. SLATER	1732 69	
WILLIAM WARNER	2213 72	
C.B. WILEY	1963 45	
H WALKER	1627 32	
FRANK McGEE	1692 18	
F.L MONTGOMERY	2138 45	
C.K. HENRY	1728 46	
TOTAL		

19. The following tabulation is a record of a week's sales in four departments of a store.

Rule a blank similar to the model.

Enter the statistics on the blank.

Find the total sales for each day made in all departments; enter these totals in the "total" column at the right.

Find the total sales made during the week in each department; enter these totals on the line at the bottom marked "total."

Find (in two ways) the total sales made in all of the departments for the entire week. Enter in the space marked Grand Total.

Find what per cent of the grand total sales was made each day; enter these per cents in the column at the right.

Find what per cent of the grand total sales was made in each department. Enter these per cents on the line at the bottom marked per cent.

One of the qualities which a business man desires in his employees is the ability to follow directions. This exercise will give you an opportunity to prove your ability along this line. Get a clear understanding of the instructions, and follow them by preparing the exercise without asking for any further directions.

A Week's Sales in a Department Store

Day	Dept. I	Dept. II	Dept. III	Dept. IV	Total	Per Cent
Monday	$362.50	$562.83	$862.94	$126.39	$	
Tuesday	415.75	475.92	732.83	143.62		
Wednesday	396.21	415.60	769.42	129.38		
Thursday	472.96	516.29	640.20	145.17		
Friday	387.29	429.36	721.32	96.27		
Saturday	493.89	562.64	816.25	163.92		
Total					Grand Total	
Per Cent						

110. To find a number when a certain per cent of it is known; that is, to find the base.

Method. Since the *percentage* is the product of two factors, the base and the rate, and since any product can be divided by one of its factors to find the other factor, it is evident that,

percentage ÷ rate = base.

Therefore, *to find the base, divide the percentage by the rate expressed as a decimal.*

Examples. 1. 18 is 25 % of what number?

Solution. 18 is the percentage, 25 % is the rate; the base is not known.
18 ÷ .25 = the required number, or base.
18 ÷ .25 = 72.

2. 45 % of the number of students in a school are boys. If there are 135 boys, how many students are in the school?

SOLUTION. $135 \div .45 = 300$, the total number of students.
This may be explained as follows:

Since 45 % of the number of students = 135,
1 % of the number of students = $\frac{1}{45}$ of 135 = 3
and 100 % of the number of students = $100 \times 3 = 300$.

Oral Work

Find the number of which:

1. 40 is 20 %.
2. 12 is 4 %.
3. 10 is 5 %.
4. 86 is 2 %.
5. \$45 is 15 %.
6. 40 is 80 %.
7. 12 is $\frac{1}{2}$ %.
8. 42 is 100 %.
9. 60 is 200 %.
10. 12 is $33\frac{1}{3}$ %.
11. 35 is 7 %.
12. 19 is $\frac{1}{2}$ %.
13. 8 is $\frac{2}{5}$ %.
14. 2700 is $66\frac{2}{3}$ %.
15. A boy lost 40 cents, which was 10 % of what he had. How much did he have?
16. A man sold 32 acres of land, which was 80 % of all that he owned. How many acres did he own?

Written Work

1. In a certain school 5 % of the pupils are absent and 475 pupils are present. What is the enrollment?
2. By saving 28 % of his salary a man saved \$2240 in 5 years. What was his salary?
3. 18.75 % of a class failed. If 26 passed, how many were in the class?
4. The distance from New York to San Francisco via the Panama Canal is 5278 miles. This distance is 60.44 % less than the distance via Cape Horn. What is the distance via Cape Horn?
5. A boy paid \$3 for a pair of shoes and had 70 % of his money left. How much money had he at first?
6. Four partners engage in business. The first invests \$4000; the second invests \$3000; each of the others invests 25 % of the total capital. What is the total capital?

7. The Washington Monument is 555 ft. and Eiffel Tower is 984 ft. high. The height of the monument is what per cent of the height of the tower?

8. The average velocity of wind is 18 mi. per hour and that of sound is 1090 ft. per second. The velocity of wind is what per cent of the velocity of sound?

9. The mean annual rainfall in Denver is 14 inches, and that in New York City is 44.8 inches. Each is what per cent of the other?

10. The length of the Hudson River is 280 miles; the length of the Ohio is 950 miles; and the length of the Mississippi is 3160 miles. The length of each is what per cent of the length of the others?

11. The record for the 100-yard dash is now $9\frac{3}{5}$ sec. It was formerly $9\frac{4}{5}$ sec. By what per cent was the record reduced?

12. In 1776 there were 13 states in the Union. There are now 48 states. What has been the per cent of increase?

13. A certain basketball player can shoot on an average 90% of his free chances. If there are 20 fouls called on his opponents in a game, how many points should he make on fouls?

14. The United States Census divides persons engaged in manufacturing into three classes: wage earners, clerks, and proprietors.

The table on page 165 shows the number of persons in the first class; that is, the wage earners. It also shows the per cent of the total number engaged in each line of manufacturing who were wage earners.

For example, there were 50,551 wage earners engaged in the manufacture of agricultural implements. The wage earners were 83.9% of the total number of persons engaged in this work? What was the total number of persons engaged in this work?

Rule a form and find the total number of persons engaged in such industries as the teacher directs.

Since the per cents are only approximate, the results will be only approximate.

Indicate the ten industries having the largest number of employees. Mark the industry with the largest number of employees "1," the second largest "2," etc.

Industry	Wage Earners Av. Number	Per Cent	Total Employees	Rank
All Industries	6,615,046	86.1		
Agricultural Implements	50,551	83.9		
Automobiles	75,721	88.7		
Boots and Shoes	198,297	91.8		
Brass and Bronze	40,618	89.4		
Bread	100,216	69.4		
Butter, Cheese	18,431	58.5		
Canning	59,968	83.3		
Carriages	69,928	84.3		
Cars and Shop Construction	282,174	93.7		
Cars, Steam	43,086	91.5		
Chemicals	23,714	85.3		
Clothing, men's	239,696	88.3		
Clothing, women's	153,743	85.9		
Confectionery	44,638	81.4		
Copper, Tin	73,615	84.7		
Cotton Goods	378,880	97.7		
Electrical Machinery	87,256	82.6		
Flour Mill	39,453	59.7		
Foundry	531,011	86.3		
Furniture	128,452	89.1		
Gas	37,215	73.0		
Hosiery	129,275	94.9		
Iron and Steel, Blast Furnaces	38,429	89.2		
Iron and Steel, Steel Works	240,076	92.1		
Leather Goods	34,907	80.2		
Leather	62,202	92.7		
Liquors, Distilled	6,430	77.2		
Liquors, Malt	54,579	81.8		
Lumber	695,019	88.5		
Marble	65,603	84.9		
Oil	17,071	80.2		
Paint and Varnish	14,240	65.0		
Paper	75,978	93.3		
Patent Medicines	22,895	55.7		
Petroleum	13,929	8.37		
Printing and Publishing	258,434	66.5		
Silk	99,037	94.1		
Slaughtering	89,728	82.5		
Smelting, Copper	15,628	92.8		
Smelting, Lead	7,424	92.1		
Sugar and Molasses	13,526	86.4		
Tobacco Manufacture	166,810	84.4		
Woolen, Worsted	168,722	96.3		
All Other Industries	1,648,441	86.0		

Percentage of Increase and Decrease

Percentage is frequently employed to find the relation between numbers, or to find how much larger or smaller one number is than another. This does not involve any new mathematical principles.

Example. What number increased by 20 % of itself is 360?

Solution. 100 % of the number = the number,
20 % of the number = the increase,
120 % of the number = 360,
1 % of the number = 360 ÷ 120 = 3,
100 % of the number = 100 × 3 = 300,

or we may say: 1.20 times the number = 360, therefore the number = $\frac{360}{1.20}$ = 300.

Oral Work

(Use equivalent fractions if more convenient.)

What number increased by

1. 20 % of itself is 240?
2. 25 % of itself is 250?
3. $12\frac{1}{2}$ % of itself is 360?
4. 200 % of itself is 1800?
5. $14\frac{2}{7}$ % of itself is 88?
6. 75 % of itself is 14?
7. 50 % of itself is 600?
8. 5 % of itself is 420?
9. 300 % of itself is 2400?
10. $37\frac{1}{2}$ % of itself is 330?
11. 1 % of itself is 202?
12. 7 % of itself is 428?
13. $6\frac{2}{3}$ % of itself is 4800?
14. $66\frac{2}{3}$ % of itself is 40?
15. 60 % of itself is 64?
16. 100 % of itself is 600?
17. How much is \$40 increased by 20 % of itself?
18. How much is 60 increased by 10 % of itself?
19. How much is \$42 increased by 10 % of itself?
20. How much is 3 increased by 100 % of itself?
21. How much is 24 bu. increased by $37\frac{1}{2}$ % of itself?
22. How much is 12 yd. increased by $33\frac{1}{3}$ % of itself?
23. How much is 10 mi. increased by 7 % of itself?
24. How much is \$18 increased by 200 % of itself?
25. How much is 32 acres increased by $12\frac{1}{2}$ % of itself?

111. Per Cent of Increase.

To find the increase, *multiply the base by the per cent of increase.*

Example. Mr. Smith, whose salary was $1200.00, received a 5% increase. How much was the increase?

SOLUTION.

$1200 Base
.05
$60.00 Increase

To find the per cent of increase, *divide the increase by the base.*

Example. Mr. Smith, whose salary was $1200.00, received an increase of $60.00. What was the per cent of increase?

SOLUTION. $60.00 ÷ $1200.00 = .05, or 5%.

To find the base.

a. Given the increase and per cent of increase.

Example. Mr. Smith received a 5% increase in salary. The increase was $60.00. What was his original salary?

SOLUTION. $60.00 ÷ .05 = $1200.00.
Increase ÷ per cent of increase = base.

b. Given the increase and the amount.

Example. Mr. Smith's salary was increased $60.00, after which he received $1260.00. What was the per cent of increase?

SOLUTION. $1260 − $60 = $1200, the base.
$60 ÷ $1200 = .05.
Hence, the per cent of increase is 5%.

c. Given the per cent of increase and the amount.

Example. After receiving a 5% increase, Mr. Smith's salary was $1260.00. What was his original salary and what was the increase?

SOLUTION. His original salary was 100% of itself. His increase was 5% of the original salary. Therefore, the new salary was 105% of the old one.

$1260 ÷ 1.05 = $1200.

Written Work

1. A merchant's sales during 1914 were $6238.92. By advertising, he increased his sales $2485.35 the following year. What was the per cent of increase, and what was the amount of the sales in 1915?

2. The deposits of the Wells Street Bank on June 30, 1914, were $365,894.56. On June 30, 1915, the deposits had increased to $396,268.55. What was the increase and the per cent of increase?

3. A factory produced 240 articles of a certain kind per day. By installing new machinery, the output was increased $12\frac{1}{2}\%$. What was the amount of the increase, and how many articles were produced per day after the new machinery was installed?

4. Williams paid $16.87 more taxes this year than he did last year; an increase of 7.8%. What was the amount of his taxes each year?

5. The payroll of the Maynard Manufacturing Company for the week ending April 4, 1914, was $428.62. The payroll for the week ending April 3, 1915, was 13% larger. What was the pay roll for the week in 1915, and what was the increase?

6. A merchant built an addition to his store which increased the floor space 28%. After the addition was completed, the floor space was 1280 square feet. What was the number of square feet of floor space in the original store, and what was the amount of the addition?

112. Per Cent of Decrease.

To find the per cent of decrease, *divide the decrease by the base.*

Example. Mr. Smith's coal bill in 1914 was $80.00. In 1915 his bill was $4.00 less. What was the per cent of decrease?

SOLUTION. Decrease ÷ base = per cent of decrease.

Thus, $4.00 ÷ $80.00 = 5%.

The decrease was 5%.

Written Work

1. The proprietor of a retail grocery employed five clerks at a total annual expense for wages of $5000.00. In order to reduce the expense, he released one clerk whose wages were $60.00 per month. How much did he decrease the cost of the clerk hire per year? What was the per cent of decrease? What was the annual cost of the clerk hire after one clerk was dismissed?

2. A paper mill which burned 2400 tons of soft coal per year remodeled its furnaces and boilers and now burns 2000 tons of coal per year. How many tons less does this factory consume per year than formerly, and what is the per cent of decrease?

3. Previous to 1914, Mr. Waterbury owned the only drug store in the town of X. When a new drug store was opened, the competition decreased Mr. Waterbury's annual sales $1067.60, or 17%. What was the amount of Mr. Waterbury's sales the year before the new store opened?

4. In 1914 Mr. Snowden, a grocer, lost $327.00 from bad debts. A credit-rating association was formed in the town; the merchants informed each other of persons who did not pay their bills; and the following year Mr. Snowden decreased his loss from bad debts 18%. How much did the credit-rating association save Mr. Snowden, and what was his loss from bad debts in 1915?

5. Rathbun moved his store into a new building where the rent was $5.00 per month less than the rent of the building which he formerly occupied. The rent in the new location was $125.00 per month. What was the per cent saved?

6. Barnes said to his partner, "We cut down our delivery expense 30% when we sold the horses and bought an auto truck. Our annual cost for making deliveries after we made the change was only $1645.00. When we delivered with horses, the cost per year was $———, so we are saving $———."

7. The following table shows one of the uses which business men make of per cent of increase and decrease.

This table compares the sales on the first Tuesday in June, 1914, with the sales on the first Tuesday in June, 1915. The merchant assumes that trade conditions were about the same on the two days. A table of this kind is prepared every day, comparing the business for the day with the business of the corresponding day of the previous year.

Prepare a table similar to the model, and enter the facts given. Show increases and per cents of increase in black ink; decreases and per cents of decrease in red ink.

What was the increase in business in department 1? What was the per cent of increase?

Complete the table.

COMPARATIVE SALES SHEET

DEPARTMENT NUMBER	SALES, TUESDAY, JUNE 2, 1914	SALES, TUESDAY, JUNE 1, 1915	INCREASE OR DECREASE	PER CENT INCREASE OR DECREASE
1	$ 826 95	$ 914 32	$	
2	1034 78	1231 64		
3	1237 62	1196 14		
4	2643 80	2843 27		
5	1413 80	1376 29		
6	962 40	1235 96		
7	2642 16	2927 92		
8	1964 39	2129 80		
9	1636 48	1596 27		
10	1213 42	1723 96		
11	1394 29	1146 92		
12	415 75	496 25		
13	3460 00	3246 29		
14	2690 70	2889 00		
15	878 25	794 60		
Total				

8. The following table shows the value of irrigating arid land. For example, the government records show that, in the regions where rainfall is insufficient, the potato crop on an acre of

irrigated land is worth $15.37 more than the crop grown on an acre of unirrigated land. This is an increase of 34.4% due to irrigation. What is the average value per acre of the potato crops grown on unirrigated land and on irrigated land?

Complete the table.

COMPARATIVE VALUE OF CROPS GROWN ON IRRIGATED AND UNIRRIGATED LAND

Crop	Excess of Average Value of Crop per Acre grown on Irrigated Land, over that grown on Unirrigated Land		Average Value of Crop per Acre on Unirrigated Land		Average Value of Crop per Acre on Irrigated Land	
Potatoes	$15	37	34.4 %			
Sugar Beets	5	39	10.4			
Wheat	8	65	58.6			
Alfalfa	5	97	35.2			
Oats	7	36	63.2			
Barley	6	51	55.1			
Corn	3	51	24.0			
Timothy and Clover, mixed	3	63	27.6			
Timothy alone	3	08	24.1			

113. Per Cent of Maximum. It is sometimes desired to compare a certain number with a larger number. For example, a certain packing plant is equipped to slaughter and pack a maximum of 2000 hogs daily. On October 22, this plant packed 1832 hogs.

The day's pack was $\frac{1832}{2000}$ of the maximum capacity, or 91.6%. State a rule for finding per cent of maximum.

Written Work

The table on page 138 shows the number of wage earners employed in coal mining in the United States in a recent year. The number of employees varies with the season. Some idea of the steadiness of employment in this industry may be formed by comparing the number of employees each month with the maximum.

During what month was the maximum number of persons employed in coal mining?

What per cent of this number was employed in January?

Complete the table.

WAGE EARNERS EMPLOYED IN COAL MINING IN THE UNITED STATES

MONTH	WAGE EARNERS EMPLOYED	PER CENT OF MAXIMUM
January	691,244	
February	686,322	
March	679,791	
April	649,870	
May	646,592	
June	652,894	
July	659,434	
August	667,146	
September	685,234	
October	704,939	
November	720,341	
December	729,273	

114. Per Cent of Average. Percentage is also used to show the relation between different numbers by comparing them with their average.

Written Work

The following table shows the monthly sales of a number of clerks:

CLERK NO.	SALES	PER CENT	CLERK NO.	SALES	PER CENT
201	$1412.32		205	$1836.24	
202	1218.16		206	1216.26	
203	967.32		207	1375.85	
204	1046.89		208	1493.85	

Find the average sales.

The sales of Clerk No. 201 are what per cent of the average?

Complete the table.

What is the value of such a table for the manager of a business?

Review Work

1. If a merchant sells all goods at an advance of 10% on the cost, what will be the selling price of an article which cost $3.40? What will be the profit on this article? If this merchant's annual sales are $24,126.35, what is his gross profit?

He marked all goods at 25% increase on the cost the following year and his sales dropped to $15,625.90. What was the cost of the goods sold, and the gross profit?

2. In 1910 the total area of the Indian reservations in the United States, exclusive of Alaska, was 77,446 square miles. In 1890 the area of such reservations was 243,991 square miles. What per cent did the area decrease in twenty years?

3. In 1910 the Indian population was 300,121, in 1890 it was 243,524. What per cent did the population increase?

4. What was the average number of acres of land in reservations per Indian for each of the years named?

5. What was the per cent of increase or decrease in the average acreage?

6. Complete the following table showing the average size of farms in different divisions of the country.

Division	Average Size of Farms (Acres)		Increase (+) Decrease (−)	
	1910	1900	Acres	Per Cent
United States	138.1	146.2		
New England	104.4	107.1		
Middle Atlantic	92.2	92.4		
East North Central	105.0	102.4		
West North Central	209.6	189.5		
South Atlantic	93.3	108.4		
East South Central	78.2	89.9		
West South Central	179.3	233.8		
Mountain	324.5	457.9		
Pacific	270.3	334.8		

7. The following table shows the amount of sugar produced in and imported to the United States. Complete the table.

	Long Tons (2240 lb.)	
Sugar imported from Hawaii and Porto Rico	718,788	
Sugar imported from other countries	1,674,776	
Total imports		x,xxx,xxx
Domestic production of cane sugar	409,960	
Domestic production of beet sugar	434,000	
Total domestic production		x,xxx,xxx
Total consumption of sugar in United States		x,xxx,xxx

a. What per cent of the total consumption of sugar was imported?

b. What per cent of the sugar consumed was produced in this country?

c. What per cent of the sugar made in this country was cane sugar? What per cent was beet sugar?

8. Complete the following table.

Class	Persons Engaged in Manufactures				
	1904		1909		Per Cent of Increase, 1904–1909
	Number	Per Cent of Total	Number	Per Cent of Total	
Proprietors and firm members	225,673		273,265		
Salaried employees	519,556		790,267		
Wage earners (average number)	5,468,383		6,615,046		
Total					

115. Percentage Analysis of a Business. Mr. E. C. Barton is the proprietor of a wholesale store. At the end of each month he analyzes the records of his business. The following analysis was made June 30, 1915. Results should be approximate to the nearest hundredth of a per cent.

1. The purchases during June, 1915, were $8146.90. Some of the goods purchased were defective, and were returned. The value of goods returned was $123.60. The returned goods were what per cent of the purchases?

2. Gross sales for June, 1915, were $11,216.29. Goods returned to the store by dissatisfied customers, $97.65. The returned sales were what per cent of the gross sales?

3. Gross sales for June, 1914, were $9862.15. Returned sales, $101.80. What per cent of the sales were returned in June, 1914?

4. What was the increase in the gross sales, and what was the per cent of increase?

5. Gross Sales − Returned Sales = Net Sales.

What were the net sales for June, 1914? For June, 1915? What was the increase in net sales, and the per cent of increase?

6. The merchandise on hand June 1, 1915, was worth $14,162.45 at selling prices. The net sales for the month were found in problem 5. What per cent of the stock on hand at the beginning of the month was sold during the month?

7. Net Sales − Cost of Goods Sold = Gross Profit.

The gross profit for June, 1914, was $1688.54. The gross profit was what per cent of the net sales for the month? (For net sales see results to problem 5.)

8. The gross profit for June, 1915, was $2001.36. The gross profit was what per cent of the net sales for that month?

9. The expenses of the business in June, 1914, were $1015.60. The expenses had increased in June, 1915, to $1140.26. What was the per cent of increase in the expenses?

10. Net Sales − Gross Profit = Cost of Goods Sold.

What was the cost of goods sold in June, 1914?

What was the cost of goods sold in June, 1915?

11. Gross Profit − Expenses = Net Profit.

What was the net profit for June, 1914?

What was the net profit for June, 1915?

12. What was the per cent of increase or decrease in the net profit?

13. Net Profit ÷ Net Sales = Per Cent of Net Profit on Sales.

What was the per cent of net profit for June, 1914?

What was the per cent of net profit for June, 1915?

CHAPTER XIII

BUYING AND SELLING MERCHANDISE

116. The Invoice. When a merchant sells goods, he usually gives the customer a bill or invoice. The invoice states the following facts:

The date of the sale.

The name and address of the seller.

The name and address of the purchaser.

The terms of the sale.

A detailed list of the articles sold, including the price of each item and the total of the invoice.

If prepaid freight is to be charged to the purchaser, the amount of freight is added to the invoice.

If the invoice has been paid, a statement of the receipt of payment is made by the seller.

Study the following invoice and state as many as possible of the facts enumerated above.

TELEPHONE, MAIN 1340
PRIVATE EXCHANGE

A. B. HUGHES

DEALER IN

STAPLE AND FANCY GROCERIES.

211-213-215 Lake Street,

Chicago, Illinois

June 10, 1915

TERMS:
2% Cash in 10 days,
Net 30 days.

SOLD TO Scobey & Company,
Fayette, Iowa

		2 Cases	Tomato Catsup	4 doz.	1.30	5	20	
		6 Cases	Elgin Canned Corn	12 doz.	1.10	13	20	
		5 Cases	Echo Peas	10 doz.	1.25	12	50	30 90

117. What the Purchaser Does. After the purchaser receives both the goods and the invoice, he inspects the merchandise to see if it agrees with the items for which he has been charged on the invoice. If stock of the correct kind and quantity is received, a check mark is entered on the invoice opposite each item in the column at the left.

The invoice is next checked to ascertain:

a. Whether the correct prices have been charged.

b. Whether any error has been made in the extension of the cost of each item, or in the total of the invoice.

118. Credit Memorandum. In case an error is found, notice is sent to the firm from which the goods were purchased. When credit is to be given by the seller on account of an error in the invoice, or for imperfect goods, or for any other cause, the purchaser will usually receive a credit memorandum, notifying him of the amount of credit entered to his account.

CREDIT MEMORANDUM

A. B. HUGHES
CHICAGO.

June 17, 19 15.

TO Williams & Co.,
Albany, New York

Your account has been credited $1.25
On account of overcharge on invoice of June 10, 1915

Written Work

Check the invoice on page 142. If the multiplications are found to be correct, enter a check mark at the right of each item. If the total is correct, check it also.

When an invoice has been checked and found to be correct, the extensions and footings are checked as shown in the illustration.

$16	50 ✓		
23	00 ✓		
4	80 ✓	$44	30 ✓

Written Work

Find the totals of each of the following sales: Apply short methods when possible.

1.

4 doz.	Men's Irish Linen Handkerchiefs	doz.	$4.43\frac{1}{2}$
5 doz.	Men's Hemstitched Handkerchiefs	doz.	5.38
14 doz.	Men's White Initial Handkerchiefs	doz.	3.23
9 doz.	Women's Imported Swiss Embroidered Handkerchiefs	doz.	$2.16\frac{3}{4}$
22 doz.	Children's Linen Handkerchiefs	doz.	$1.23\frac{1}{2}$
7 doz.	Women's Swiss Lawn Handkerchiefs	doz.	$.67\frac{1}{2}$
13 doz.	Children's Mull Handkerchiefs	doz.	$.72\frac{1}{2}$

2.

17 doz.	Men's Leather Gauntlets	doz.	$ 7.68
11 doz.	Men's Buckskin Gloves, sizes 8 to 10	doz.	9.49
16 pairs	Men's Leather Automobile Mittens	pair	$1.36\frac{1}{2}$
$4\frac{1}{2}$ doz.	Men's Suede Gloves, sizes $7\frac{3}{4}$ to 10	doz.	4.44
$3\frac{1}{2}$ doz.	Men's No. J 264 Kid Gloves	doz.	10.50
$5\frac{1}{2}$ doz.	Imported Kasan Cape Gloves	doz.	13.50

3.

16	Silk Taffeta Umbrellas	each	$2.15
28	8 Rib Pure Silk Taffeta Umbrellas	each	4.25
32	Men's Suit Case Umbrellas	each	2.25
12	Extra Size Umbrellas	each	$.87\frac{1}{2}$

4.

3 doz. pairs	Walton Cotton Blankets	pair	$ $.82\frac{1}{2}$
1 doz. pairs	Cotton Crib Blankets	pair	$.57\frac{1}{2}$
8 doz. pairs	Wool Filled Gray Blankets	pair	3.85
7 doz. pairs	Douglass Wool Finish Plaids	pair	$1.89\frac{1}{2}$
4 doz. pairs	All Wool Plaids	pair	6.00
10 doz. pairs	"Canada Camp" Blankets	pair	$1.12\frac{1}{2}$

5.

20	Assorted Pattern Smyrna Rugs, 36 × 72	each	$2.25
80	Scotch Jute Rugs	each	$.37\frac{1}{2}$
3 doz.	Cotton Bathroom Rugs, 27 × 54	each	$1.12\frac{1}{2}$

6.

3 doz.	Size 6	Children's Fast Black Hose	doz.	$1.17½
2 doz.	Size 6½	Children's Fast Black Hose	doz.	1.21½
4 doz.	Size 7	Children's Fast Black Hose	doz.	1.24½
5 doz.	Size 7½	Children's Fast Black Hose	doz.	1.27½
6 doz.	Size 8	Children's Fast Black Hose	doz.	1.31½
8 doz.	Size 8½	Children's Fast Black Hose	doz.	1.42½
7 doz.	Size 9	Children's Fast Black Hose	doz.	1.49¼
6 doz.	Size 9½	Children's Fast Black Hose	doz.	1.61
5 doz.	Size 10	Children's Fast Black Hose	doz.	1.66½

7.

20 No. B 645	Silk Woven Waist Patterns (each pattern 3½ yd.)	yd.	$.16⅔
25 No. B 493	Dresden Silk Mull Waist Patterns (3½ yd. pieces)	yd.	.21½
18 bolts	Irish Linette, 10, 11², 12, 10³, 11¹, 11³, 12¹, 11³, 10³, 12, 10, 10³, 10¹, 11², 12, 11³, 10², 10³	yd.	.14½
20 bolts	French Gauze Chiffon, 14, 13³, 12³, 14¹, 14, 13², 13, 12³, 13, 13¹, 14², 13³, 14², 14¹, 13³, 12³, 14, 13³, 13², 12³	yd.	.37½

Note. The small figures mean quarter yards. Thus 11² means 11¼ yards.

Explanation of Grocery Orders

The sign # if placed before figures means "number"; if placed after figures it means pounds.

Articles sold in cases [cs.] are usually priced by the dozen. The total number of dozens in the cases is therefore given after the name of the commodity, and the price is the price per dozen.

Some articles sold in barrels and cases are priced by the pound; in such instances the total number of pounds in all the barrels or cases appears immediately before the price.

Sugar is sold by the hundred pounds (cwt.). The net weight of each barrel is given. Find the total number of pounds; point off two places to the left to find the number of hundred pounds, and multiply by the price per hundred.

8.

		Price
7 cases Acme Peas	14 doz.	$1.40
3 boxes Peona Soap		4.95
7 bbl. Northern Salt		2.10
15 bbl. Winter Wheat Flour $\frac{1}{8}$ sacks		7.10
12 bbl. H. & E. Granulated Sugar 329, 335, 347, 351, 344, 347, 350, 331, 342, 355, 349, 333		6.15 per cwt.
5 sacks H. & E. Granulated Sugar	500#	6.10 per cwt.
9 cases Acme Peas	18 doz.	1.45
17 cases Acme Corn	34 doz.	1.70
6 boxes Dried Apples	150#	.08$\frac{3}{4}$
2 bbl. 10# sacks Northern Salt		1.95
6 cases Algonac Tomatoes	12 doz.	1.37$\frac{1}{2}$

9.

1 cs. 25# Macaroni		2.10
5# Cream of Tartar		.40
1 cs. Acme Peas	2 doz.	1.30
5 cases Hawthorne Pears	10 doz.	2.95
11 cases Hawthorne Peaches	22 doz.	2.50
12 cans Pimento		.14
15 cs. H.L. Shredded Pineapple		2.40
10 cases Tall Salmon	40 doz.	2.10
12 cases F. & B. Tomatoes	24 doz.	3.75
85 sacks Yellow Corn Meal	850#	.03
8 cases 25# Macaroni		2.10
5 cases 15# Spaghetti		2.10
7$\frac{1}{2}$ doz. pkgs. Seeded Raisins		1.40
125# Rice		.07
25 quarts Olive Oil		.75 gal.
18$\frac{3}{4}$ gal. Sweet Pickles		1.06$\frac{1}{4}$
6$\frac{1}{2}$ gal. Olives $\frac{70}{80}$		1.30
25 cases 4 oz. Grd. Pepper	125#	.30
6 cans Paprika		2.25 doz.
14 cases G. M. Soap		2.85

		Price
12 cases Soda Crackers	$91\frac{3}{4}$#	$ .09
12 Cracker cans		.50
1 tub Peanut Butter	$48\frac{1}{2}$#	.$12\frac{1}{2}$
$62\frac{1}{2}$ bbl. $\frac{1}{8}$ sacks Spring Wheat Flour		5.80
$49\frac{3}{4}$ bbl. $\frac{1}{8}$ sacks Winter Wheat Flour		6.25
16 cases #2 Algonac Lima Beans	32 doz.	1.40
14 cases #2 Algonac String Beans	28 doz.	1.$37\frac{1}{2}$

10.

10 doz. W/W 6 in. Plates	.82 doz.
$13\frac{1}{2}$ doz. W/W 4 in. Plates	.$62\frac{1}{2}$ doz.
5 doz. /S/ $2\frac{1}{2}$ in. Butter Chips Tk.	.22 doz.
$8\frac{1}{4}$ doz. #13 Cups	.$87\frac{1}{2}$ doz.
6 doz. W/W 4 in. Ice Creams	.44 doz.
15 doz. #64 /G/ Nappies	1.$12\frac{1}{2}$ doz.
8 doz. A.C. $\frac{1}{2}$ Tk. 30's L/F Bowls	1.28 doz.
$\frac{3}{4}$ doz. #211 Tea Pots	1.25 doz.
$2\frac{1}{2}$ doz. #0 /S/ Custard Pots	.98 doz.
17 doz. #1352 Tumblers	.45 doz.
12 doz. #1296 Finger Bowls	1.$33\frac{1}{3}$ doz.
$1\frac{3}{4}$ doz. #3234 Oil Bottles	2.25 doz.
$3\frac{1}{2}$ doz. 12–199 Salts	1.50 doz.
$3\frac{1}{2}$ doz. 12–200 Peppers	1.50 doz.
$2\frac{1}{6}$ doz. 444 A Pitchers	5.00 doz.
$\frac{1}{3}$ doz. Punch Glasses	.75 each
$12\frac{1}{2}$ doz. 3404 Dessert Forks	2.35 doz.
$12\frac{1}{2}$ doz. 3405 Dessert Knives	2.30 doz.
16 doz. 3413 Tea Spoons	1.$37\frac{1}{2}$ doz.
8 doz. 3406 Dessert Spoons	2.25 doz.

Rule invoices and enter the following sales, assuming that you are the selling merchant.

11. To W. K. Sears,
Elgin, Illinois.
Sept. 10.

45 No. R 721 Body Brussels Rugs	6 × 9 ft. each	$13.25
32 No. R 731 Body Brussels Rugs	8 × 10 ft. each	19.75

			Price
36	No. R 741	Body Brussels Rugs 9 × 12 ft. each	$22.87½
25	No. R 751	Body Brussels Rugs 11 × 13 ft. each	30.80

12. To C. B. Perkins,
Amherst, Minn.
Sept. 12.

20	H 20	Family Coffee Mills	each	$1.35
16 sets	H 33	Sad Irons	set	.73
12 doz.	H 545	Flour Sifters	doz.	1.83⅞
25 doz.	H 463	Carpet Beaters	doz.	.89
8 doz.	H 6646	Galvanized Clothes Lines, 20 gauge wire,	doz.	.92⅜
10 doz.	H 6546	Galvanized Clothes Lines, 18 gauge wire,	doz.	1.52½

13. To Owen & Hendersen,
Cedar Rapids, Iowa.
Sept. 13.

35	F 384	Plush Library Chairs	each	$14.75
28	F 376	Reed Rockers	each	3.10
18	F 394	Bedroom Rockers	each	4.85
26	F 465	Roman Chairs	each	7.88
19	F 414	Morris Chairs	each	18.95

CHAPTER XIV

COMMERCIAL DISCOUNTS

Cash Discount

119. Purpose of Cash Discount. In order to encourage prompt settlement of accounts, merchants frequently offer to deduct a certain per cent of the bill if it is paid within a fixed number of days. This deduction for prompt payment is called **cash discount.**

120. Terms. The terms state the discounts offered and the time when the bill is due. The terms are often expressed by an arrangement similar to the following:

2/10; 1/30; N/60.

The figures at the left of the line indicate the rate of discount offered; the figures at the right indicate the number of days within which payment must be made in order to obtain the discount. Thus, the terms stated above mean:

2 % discount from the face of the bill, if paid in 10 days;

1 % discount from the face of the bill, if paid in 30 days;

Net Amount (no discount is allowed, and bill is due) in 60 days.

121. Definitions. The amount of the purchase *before* subtracting the discounts is called the **list price.**

The amount of the purchase after subtracting the discounts is called the **net price.**

The **Rate of Discount** is stated as a per cent of the list price.

122. Cash Discount Illustrated. The terms of the invoice on page 142 were 2/10; N/30.

In order to obtain the discount, Scobey & Co.'s payment must be made on or before June 20, 1915.

The bill is due and payment is expected *in full* on July 10, 1915.

No discount is allowed if payment is made between June 20 and July 10, 1915.

123. Computing the Discount and the Net Price.

First Method:

$95.25	List price
.02	Rate of discount
$1.9050	Discount

$95.25	List price
1.91	Discount
$93.34	Net price

State a rule for finding discount and net price by this method.

(Note that five mills or more in the discount are considered a cent. Thus, $1.905 is considered $1.91.)

Second Method:

100%	List price
2%	Rate of discount
98%	Per cent of bill to be paid

$95.25	List price
.98	
$93.345	Net price
or $93.34	

State a rule for computing the net price by this method.

124. The Advantages of Cash Discount. Cash Discounts may benefit both the purchaser and the seller. Merchants offer cash discounts because they encourage prompt payments, and thus decrease:

a. the loss from bad debts;

b. the cost of collecting accounts;

c. the amount of capital tied up in outstanding accounts.

It is usually good business for the purchaser to "take his discount" (pay the bill before it is due), even though the rate of discount may be small. An illustration will show this fact.

Suppose the terms of the sale are, Cash less 1%; net 30 days. If the purchaser pays cash, he receives 1% for the use of his money for one month. 1% a month is 12% a year; a high rate of interest.

The rates of cash discount are usually small, varying from 1% to 5%.

Small discounts are offered if the bill is due in a short time.

For example, 1/10; N/30.

Larger discounts are offered if the bill is due after a greater length of time.

For example, 5/30; N/4 months.

As a further means of insuring payment of invoices, merchants frequently charge interest on bills which are not paid when due.

Written Work

Find the list price, the discount, and the net price of each of the following purchases, and answer the following questions about each invoice.

a. What is the last day on which payment can be made and the discount secured?

b. When is the invoice due?

1. Invoice dated April 4. Terms 2/10; N/30.

7 cases Acme Peas	$1.40
3 cases Osea Soap	4.95
7 bbl. Northern Salt	2.10
15 bbl. ⅛ sacks A. D. Flour	7.10

2. Invoice dated October 26. Terms 1/15; N/30.

12 bbl. H. E. Granulated Sugar, 329, 335, 347, 351, 344, 347, 350, 331, 342, 355, 349, 333 #	$6.15
5 sacks H. E. Granulated Sugar, 500 #	6.10

3. Invoice dated January 5. Terms 2/10; 1/20; N/30.

16 bolts White Dress Linen, 10, 12, 11^1, 12^2, 10^3, 11^3, 10^3, 12, 10^1, 11^2, 12, 11^1, 10^2, 11^3, 12^1, 10^2	yd.	$.44\frac{1}{4}$
12 bolts Persian Lawn, 24^3, 25^1, 23^2, 25, 24^2, 24^3, 25^2, 24^1, 23^3, 24^3, 25^2, 25	yd.	$.17\frac{3}{4}$
48 bolts French Nainsook, 12-yard pieces	yd.	$.18\frac{1}{4}$
36 bolts Mercerized Lingerie Batiste, 24-yard pieces	yd.	.21
24 bolts Imported Linen Lawn, 10-yard pieces	yd.	$.43\frac{3}{4}$

NOTE. There will be two possible net prices for this invoice.

4. Complete the following table.

Combine the dollars in the column at the left, with the cents in the row near the top, to form the list price. Deduct the discount shown above the cents, and enter the net price in the table. The two results entered in the table show the method. Thus,

a.	$26.10	the list price	*b.*	$641.67	
	.02	the discount		.025	
	$ .52	the discount at 2%		$ 16.04	the discount at 2½%
	$26.10			$641.67	
	.52			16.04	
	$25.58	Net price		$625.63	Net price

	Less 2%	Less 5%	Less 3%	Less 2½%
	10	45	75	67
$26 00	$25 58			
14 00				
73 00				
19 00				
261 00				
112 00				
317 00				
216 00				
641 00				$625 63
5 00				
24 00				
43 00				

Trade Discount

125. Purpose of Trade Discount. In some lines of business merchants sell both at wholesale and retail. They advertise their goods at a certain price, but when they sell to dealers, they frequently deduct a part of this price. The amount deducted is called a **Trade Discount.**

Trade discounts are most common in businesses which issue catalogues. When the catalogue is sent to a dealer, a discount sheet is inclosed.

Specimen Discount Sheet

DISCOUNT SHEET

The following discounts are offered on articles listed in catalogue No. A 23.

Pages 1 to 15	20 %
Pages 16 to 39	28 %
Pages 39 to 67	40 %
Pages 68 to 90	Net
Pages 91 to 136	14⅔ %

Do not show this discount sheet to your customers.

NOTE. "Pages 68 to 90 Net" means, no trade discount is offered for goods listed on these pages.

Not Responsible for Goods Lost or Damaged in Transit. Claims for Allowance must be made upon Receipt of Goods. Address all Communications to Abercrombie & Co., Chicago

ABERCROMBIE & COMPANY

PUBLISHERS & BOOKSELLERS

245 WABASH AVENUE

CHICAGO, March 3, 19 15

SOLD TO W. M. Rickert Co.,
Home,
Iowa

YOUR ORDER NO. 121
TERMS, 2/10; N/30
CONVEYANCE, U. S. Ex.

IN REFERRING TO THIS ORDER MENTION NO. 4336 ENTERED W. B. 3/3/15.

12	Copies H. & J. Second Year English	.85	10	20		
21	" Beeman's Algebra	.90	18	90		
			29	10		
	Less 18%		5	24	23	86

Class Discussion

1. Why do merchants offer trade discounts?

2. What is the catalogue price of the H. & J. Second Year English per copy?

3. How much does each copy of the English book actually cost the retail dealer?

4. If Mr. Rickert sells this book for the catalogue price, 85 cents, how much profit does he make on each book?

5. Trade discounts are usually larger than cash discounts. Why do you think this is the case?

126. Computing Trade Discounts. Trade discounts are computed in the same manner as cash discounts.

Example. What will goods cost a dealer if they are sold to him for \$46, less a trade discount of 25%?

SOLUTION.

\$46.00	List price	\$46.00	List price
.25	Rate of discount	11.50	Trade discount
\$11.50	Trade discount	\$34.50	Price to dealer

Written Work

Complete the following table. Enter the price to dealer after deducting the Trade Discount.

TRADE DISCOUNT	CATALOGUE PRICE					
	\$27	85	\$126	39	\$96	45
$14\frac{2}{7}$%						
18%						
20%						
22%						
25%						
15%						
17%						
28%						
$16\frac{2}{3}$%						
10%						

127. Series of Two Discounts. Invoices are often subject to a cash discount in addition to the trade discount. Two or more discounts are called a "Discount Series."

Example. What is the smallest amount of money that will pay a bill of $236, subject to a trade discount of 20 % and a cash discount of 2 % ?

SOLUTION.

$236.00	List price	$188.80	Trade price
.20	Rate of trade discount	.02	Rate of cash discount
$47.20	Trade discount	$3.7760	Cash discount
$236.00	List price	$188.80	Trade price
47.20	Trade discount	3.78	Cash discount
$188.80	Price to the trade	$185.02	Net price

The cash discount is computed on the trade price, *not* on the list price. Therefore, 20 % trade discount and 2 % cash discount are not the same as a single discount of 22 %.

Written Work

Complete the following table. From each list price deduct the trade discount indicated above the list price; and from the trade price thus found, deduct the cash discount shown at the left.

CASH DISCOUNT	FACTS TO BE FOUND	TRADE DISCOUNT, 28 %	TRADE DISCOUNT, 25 %	TRADE DISCOUNT, 33⅓ %
2 %	List Price	$36 45	$68 40	$239 64
	Trade Discount			
	Trade Price			
	Cash Discount			
	Net Price			
3 %	List Price	205 08	333 95	47 30
	Trade Discount			
	Trade Price			
	Cash Discount			
	Net Price			
5 %	List Price	35 00	713 20	609 49
	Trade Discount			
	Trade Price			
	Cash Discount			
	Net Price			

Quantity Discounts

128. Purpose of Quantity Discounts. In some lines of business, particularly in manufacturing, it is customary to give a larger rate of discount on a large order than on a small one.

A mail order firm advertises:

1 % discount on an invoice of \$10; 2 % on \$20; 5 % on \$35.

What is the purpose of such quantity discounts?

A printer advertised the following rates:

\$10 per thousand for the first thousand copies. On an order of more than 1000 copies, a discount of 15 %. This discount is offered because the work of typesetting, making up the forms, and making ready the press must all be done, although only one copy is to be printed. The cost of printing 2000 copies, therefore, is not twice as much as for 1000 copies.

Written Work

1. The following prices were quoted by a manufacturer of lockers: No. 6094. Each \$6.30.

Discounts:

Orders of from 50 to 100 — Less 8 %.
Orders of from 101 to 300 — Less 10 %.
Orders of from 301 to 500 — Less 12½ %.

Find the total cost, and the cost per locker of an order of 75 lockers; 140; 475.

2. A paper manufacturer quoted the following prices:

No. 020 note paper, cut to size 8 by 10, \$.55 per ream.

An extra charge of 3 cents per ream will be made for wrapping in packages of 500 sheets. (500 sheets considered 1 ream.)

On orders of 100 reams or more a discount of 4 % will be allowed.

Terms, 1/10; N/30.

Find the cost and state what discounts were allowed:

a. An order of 50 reams, unwrapped, payment made 20 days from date of sale.

b. An order of 150 reams, unwrapped, payment made 15 days from date of sale.

c. An order of 150 reams, wrapped, payment made 18 days from date of sale.

d. An order of 240 reams, wrapped, payment made 7 days from date of sale.

e. An order of 60 reams, wrapped, payment made 8 days from date of sale.

Fluctuation Discount

129. Purpose of Fluctuation Discounts.

Another important discount is that offered to change the catalogue price of an article to meet the changes, or fluctuations, in the market price. Fluctuation discounts are used by establishments which sell goods made from raw material the price of which frequently changes.

For example, suppose an article made from steel (the market price of which varies at frequent intervals) is quoted in the catalogue at \$8.

When the market price of steel drops and the article can be sold for \$7, a discount of $12\frac{1}{2}$ % can be offered.

If raw steel should drop in price so that the article could be sold for \$6, a discount of 25 % could be offered.

The fluctuation discount sheet is a means of economy to the manufacturer, because he can issue new discount sheets much more cheaply than complete catalogues.

130. Computing Fluctuation Discounts.

Example. What discount will change a catalogue price of \$3 to a market price of \$2.50?

Solution.

\$3.00	Catalogue price
2.50	Price at which article is to be sold
\$ .50	Amount to be deducted by discount

$\$.50 \div \$3.00 = 16\frac{2}{3}$ %, rate of discount to be offered

State a rule for finding the rate of discount by this method.

When the quotient obtained by the division involves a fractional per cent, the next lower whole number is sometimes taken as the per cent. Thus, a quotient of $15\frac{1}{7}$ % may be regarded as 15 %, a discount of 18.27 % may be regarded as 18 %.

Example. An article is listed at $24.50. The market conditions are such that it should sell for about $21.30. What rate of discount should be offered?

SOLUTION. $24.50 List price
21.30 Market price
$ 3.20 Amount to be deducted by discount
$3.20 ÷ $24.50 = 13.06 %.
13 % would therefore be offered.
This discount would make the net price $. ?

Written Work

Complete the following table, showing what rate of discount would be offered to reduce the list prices at the left to the market prices shown at the top of the table.

Express results to the nearest per cent.

List Prices	Market Prices				
	$8.50	$7.75	$8.00	$7.60	$6.00
$10.00					
12.00					
11.50					
9.80					
10.50					

Discount Series

131. Trade and Fluctuation Discounts Combined.

Some business concerns offer an unchanging trade discount and a changing fluctuation discount. This slightly increases the task of determining what fluctuation discount to offer.

Example. An article is listed at $12 less a trade discount of 25 %. The market value drops so that the real net price should be $6. What additional discount must be offered?

SOLUTION. $12 × .25 = $3, Trade discount.
$12 − $3 = $9, Price less trade discount.
$9 − $6 = $3, amount to be deducted by 2d discount.
$3 ÷ $9 = 33⅓ %, rate of fluctuation discount.

Written Work

Complete the following table.

The columns at the left show the list price and the trade discount offered. Find what fluctuation discount must be offered to reduce the price to market values.

Approximate results, correct to the nearest hundredth of a per cent.

List Prices	Trade Discount	Market Values			
		$24 00	$23 50	$22 00	$20 00
$28 00	10 %				
33 00	15				
30 50	12				
40 00	28				

Written Review

1. An implement dealer purchased the following invoice:

Dated April 17.

4 No. 364 Plows, $38.75, less 20 %.

7 Self Dump Hay Rakes, $ 20.50, less 18 %.

7 No. 264 Hay Stackers, $41.50, less 15 %.

Terms, 2 % for Cash in 20 days. Net 90 days.

On May 2, he paid the bill with a check for $——.

2. Which of the following prices is better for the purchaser:

$45, less 25 %, 18 %, and 2 %;

$60, less 28 %, 25 %, and 1 %?

3. A merchant's discounts were 25 % and 15 %. A clerk sold an invoice of $72 and gave a single discount of 40 %. How much did his error cost his employer?

4. A merchant lists a desk at $45 less 20 %. A competitor sells a similar desk for $48 less $33\frac{1}{3}$ %. In order to exactly meet his competitor's price, the first merchant decides to give an additional discount of —%.

5. Hewett paid an invoice in time to secure a discount of 3 %. If the check sent was for $208.55, what was the list price of the invoice?

6. Graff Brothers sent a check to a wholesale house to pay an invoice. The check was for $801.90. What was the list price of the invoice if the discounts taken were 10 % and 1 %?

132. A Short Method of Finding a Single Discount Equivalent to a Discount Series. A single discount equivalent to two discounts may be found as follows:

From the sum of the two discounts subtract their product.

The rule may be easier to remember if stated thus:

Add the two discounts; multiply the two discounts; subtract the second result from the first.

Example. What single discount is equivalent to a trade discount of 20 % and a cash discount of 2 %?

SOLUTION.

$$.20 + .02 = .22$$
$$.20 \times .02 = .004$$
$$\text{Single discount} = .216 \text{ or } 21.6\,\%$$

The buyers in some large business houses prepare an elaborate table similar to the one which follows. When manufacturers quote prices subject to a discount series, the buyers can tell by a glance at the table what single discount the series equals.

Written Work

Complete the following table, using the method just explained to find single discounts equivalent to the discount series.

TRADE DISCOUNT	CASH DISCOUNT, 3 %	CASH DISCOUNT, 6 %	CASH DISCOUNT, 5 %	CASH DISCOUNT, 7 %	CASH DISCOUNT, 10 %
18 %					
15					
20					
25					
$33\frac{1}{3}$					

With the aid of the table, find the net price of the following:

a. An invoice of \$215, less 18 % and 5 %.

b. An invoice of \$464.20 less 18 % and 10 %.

c. An invoice of \$12.40 less 20 % and 6 %.

d. An invoice of \$23.87 less $33\frac{1}{3}$ % and 3 %.

133. Series of Three Discounts.

It sometimes happens that a bill is subject to several discounts. Explain how this might be the case.

To find the net price of a bill subject to three discounts, find the discount equivalent to two of the discounts stated, then find the discount equivalent to this result and the third discount.

Example. What discount is equivalent to three discounts 25 %, 20 %, 10 %?

SOLUTION. First find the discount equivalent to discounts of 25 % and 20 %; the result is 40 %.

Find the discount equivalent to discounts of 40 % and 10 %; the result is 46 %. Therefore a discount of 46 % is equivalent to discounts of 25 %, 20 %, 10 %.

Written Work

Find the net cost of the following bills:

1. \$45 less discounts of 20 %, 15 %, and 10 %.
2. \$28 less discounts of 25 %, 10 %, and 5 %.
3. \$70 less discounts of 30 %, 20 %, and 2 %.

Find a single discount equivalent to each of the following series:

4. 20 %, $12\frac{1}{2}$ %, and 8 %.
5. 15 %, 10 %, and 4 %.
6. 28 %, 14 %, and 2 %.
7. 25 %, 20 %, and 10 %.
8. 40 %, 20 %, and 10 %.
9. $33\frac{1}{3}$ %, 20 %, and $12\frac{1}{2}$ %.
10. $37\frac{1}{2}$ %, 10 %, and 20 %.

CHAPTER XV

RECORDING PURCHASES AND SALES

134. The Purchases Book. Merchants usually keep a record of purchases and sales. There are several different kinds of books used for this purpose. The following illustration shows a common form of the Purchases Book.

PURCHASES BOOK

Date of Invoice	From Whom Purchased	Amount	Terms	Discount term Expires	Cash Discount	Due Date	When & How Paid
1915							By Check
March 3	Abercrombie & Co.	23 86	2/10 N/30	Mar 13	48	Apr 2	Mar 12 $23.38

The entries in the purchases book are made from the invoice received at the time of the purchase. The model shows proper record of the purchase made from Abercrombie & Co., as shown by the invoice on page 153.

Notice that $23.86 is the amount of the invoice after deducting the trade discount of 18 %. The cash discount is not deducted until the bill is paid.

The terms are taken from the invoice.

The discount term expires March 13. It is important to have this date recorded in the purchases book, as it is the last date on which payment may be made and the discount secured.

April 2 is the day when the bill is due and payment is expected. No discount is allowed when payment is made between March 13 and April 2.

135. How to find the Date when an Invoice is Due. If the terms are stated in days, count the actual number of days. Thus, 30 days from March 3 is April 2.

If the terms are stated in months, calendar months are counted. Thus, if the terms had been 2/10; N/1 month, the invoice would have been due on April 3.

No entry is made in the "When and How Paid" column until the invoice is paid.

It is not necessary to enter the items in the purchases book, because the invoice can be kept on file to supply this information.

136. The Sales Book. The following illustration shows a page from a commonly used form of sales book.

The entries were made from the invoice on page 153.

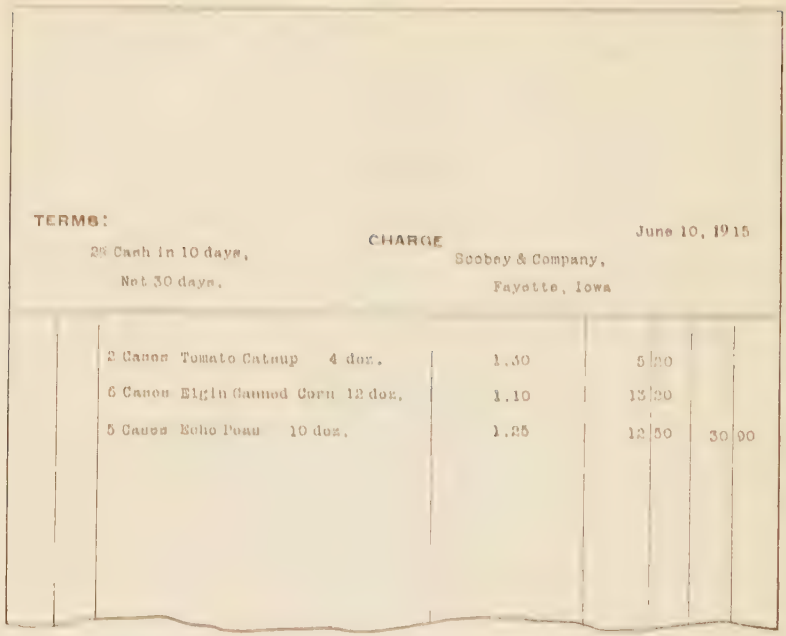
March 3, 1915

		W. M. Rickert Co. Home, Iowa					
		Terms 2/10, N/30					
		12 Copies H. & J. Second Year English	85	10	20		
		21 " Beeman's Algebra	90	18	90		
				29	10		
		Less 18%		5	24	23	86

137. Loose-leaf Sales Book. Many merchants record their sales in a loose-leaf sales book. At the time the invoice is made, a carbon copy is also made. This requires very little extra labor, and the carbon copies, called "Charge Sheets," having holes punched at the side, can be bound together in a binder.

The loose-leaf sales book has several advantages. Both the invoice and the charge sheet, which forms the sales book, can be made on the typewriter at the same time; and fraud is prevented because the invoices are numbered, and a clerk cannot sell goods and make an invoice without also making a charge slip. He cannot keep the money and destroy the charge slip because one of the numbered sheets would be missing, and the fraud would be apparent.

The following illustration shows the charge sheet made as a carbon copy of the invoice shown on page 142.



TERMS:
2% Cash in 10 days,
Net 30 days.

CHARGE
Scobey & Company,
Fayette, Iowa

June 10, 1915

2 Canon Tomato Catsup 4 doz.	1.50	5	20	
6 Canon Elgin Canned Corn 12 doz.	1.10	13	20	
5 Cases Echo Peas 10 doz.	1.25	12	50	30 90

Written Work

Rule a Purchases Book and record the following purchases making proper extensions.

1. From Eaton & Dunham, 213 Main Street, Indianapolis, Indiana.

Date, June 5, 1915.
Terms, 1/15; N/60.

6 doz. Silk Four-in-hand Ties	each	$	.37½
3 doz. White Lawn Ties	gross		2.94
38 doz. Assorted Style Amoryth Linen Collars	doz.		1.92
18 doz. Policeman's Suspenders	doz.		4.37½
9 doz. Khaki Overalls	doz.		8.97
6 doz. Boys' Flannelette Waists (ages 4 to 13 yr.)	doz.		2.16¼
10 doz. Men's Flannel Work Shirts (sizes 14 to 18)	doz.		8.76½
2¼ doz. Men's Negligee Shirts	doz.		16.40

2. From J. B. Clark, Aurora, Illinois.
Date, June 8, 1915.
Terms 2/20; N/3 months.

7 doz.	Assorted Sizes Men's Silk Half Hose	doz.	$4.29
19½ doz.	Mercerized Lisle Hose	doz.	4.78
3 doz.	Men's Black Overgaiters	doz.	6.93
18	Women's Taffeta Silk Waists (sizes 34 to 42)	each	2.17
24	Navy Blue Silk Chiffon Waists	each	2.98
5 doz.	Embroidered Lace Coat Sets	doz.	6.93¾
2 doz.	Amoskeag Gingham Aprons	doz.	4.69½
2¼ doz.	Lace Jabots	doz.	4.25

3. From Bishop & McGee, Independence, Iowa.
Date, June 10, 1915.
Terms, 1/5; N/20.

10 doz.	Defiance Food Choppers	each	$2.85
9 doz.	Climax Food Choppers	each	.72
14 doz.	No. 7 size Skillets	each	.22½
30 sets	Enamel-lined Iron Kettles, each set containing		
	3 2 quarts at .21		
	4 4 quarts at .29¾		
	6 6 quarts at .34⅔		
	4 8 quarts at .39¾		
	2 10 quarts at .46⅔		
2¾ doz.	H 921 Waffle Irons	doz.	$7.84⅞
6½ doz.	H 922 Waffle Irons	each	.93
25	H 464 Soapstone Griddles (round)	each	.55
12 doz.	H 465 Soapstone Griddles (oval)	each	.65⅜

4. From A. D. McHaughton, Fairchild, Missouri.
Date, July 14, 1915.
Terms, 3/30; N/90.

5 doz.	H 731 Wire Waste Baskets	doz.	$ .89
15 doz.	H 732 Wire Letter Baskets	doz.	1.23
28	H 881 Spring Seats	each	.88

15 sets	H 393 Cobblers' Outfits	set	$	.96
19 doz.	H 242 Curry Combs	doz.		.67
14	F 272 Upholstered Rockers	each		8.75
25	F 279 Oak Rockers	each		12.65
20	F 212 Turkish Rockers	each		12.75

Rule a Sales Book, and record the following sales, making proper extension.

5. To J. D. Preston, Monmouth, Illinois.
July 7, 1915.
Terms, 1/10; N/40.

47 No. L 601 Royal Worsted Wilton Rugs	8 × 10 ft.	each	$29.25
52 No. L 602 Royal Worsted Wilton Rugs	6 × 7 ft.	each	19.75
38 No. L 603 Royal Worsted Wilton Rugs	9 × 12 ft.	each	32.35
65 No. L 661 Worsted French Wiltons	9 × 12 ft.	each	44.60
50 No. L 662 Plain Color Wiltons	8 × 10 ft.	each	31.75

6. To R. J. Noble, Dubuque, Iowa.
July 8, 1915.
Terms, 1/15; N/60.

25 doz.	Half-bleached Cotton Towels	24 × 54	doz.	$ .98
16 doz.	Linen Monogram Towels	22 × 39	doz.	4.95
20 doz.	Bleached Turkish Bath Towels	23 × 52	doz.	2.37½
14 doz.	No. T 291 Cotton Face Cloths	10 × 13	doz.	.42½

7. To Oscar Hamilton, Reynolds, North Dakota.
July 9, 1915.
Terms, 2/5; N/2 months.

15 doz.	Palmetto Fiber Scrub Brushes		doz.	$ .86
22 doz.	H 221 Kitchen Spoons, 10 inch		doz.	.23
19 doz.	H 241 Kitchen Spoons, 12 inch		doz.	.27
26 doz.	H 251 Kitchen Spoons, 14 inch		doz.	.33
38 doz.	H 333 Kitchen Forks, 13 inch		doz.	.19
36 doz.	H 334 Kitchen Forks, 15 inch		gross	3.20
12 doz.	H 341 Perforated Steel Spoons		doz.	.84⅞
8 doz.	H 691 Kitchen Sets		doz.	8.75
16 doz.	H 692 Kitchen Sets		doz.	3.95

Marking Goods

138. Method of Marking Goods. When stock is placed on the shelves in the salesroom, the cost of each article should be marked either on the goods, on the package which contains them, on tags attached to the goods, or on card lists placed near the goods.

139. Advantage of Marking. In some lines of business it is necessary to have the selling price, or both the cost and the selling price, marked on the goods. When the stock becomes low and the buyer wishes to purchase a new supply, he can compare the price paid for the goods on the shelves, with quotations of prices made to him by the salesmen from the wholesale houses. It would also be a convenience to know the cost of an article if it proved to be a slow seller, and the manager determined to sell at a reduced price to unload the stock. One of the chief advantages of cost marking is in taking inventory of stock and finding its value at cost prices. In marking goods, the cost price is taken from the invoice.

140. "Blind Price Lists." It would be unwise to mark the cost in figures, as this would disclose the cost and the profit to purchasers. It is, therefore, customary for merchants to adopt a set of symbols, called a cipher, or blind price list. Any symbols may be chosen, but they will be more easily written if letters are used, and these letters will be more easily remembered if they form a word or phrase. The word or phrase selected must not contain the same letter twice. Otherwise the same letter will represent two different numbers.

The following will illustrate :

a d m o n i s h e r	mah
1 2 3 4 5 6 7 8 9 0	318
F i t z a u b r e y	izy
1 2 3 4 5 6 7 8 9 0	240

141. The Repeater. To further conceal the cost a "repeater" should be adopted. When the same figure is repeated, as in $1.55, the repeater sign is used for the repeated figure. "x" is frequently

used as a repeater, but because it is so commonly used, some other repeater would perhaps be preferable. Words or phrases with eleven letters are frequently chosen as keys, one of the letters being used as a repeater.

Using "t" as the repeater, and "admonisher" as the key,

$3.88 would be written *mht.*

Written Work

Using "blacksmith" as the key word, and "d" as the repeater, indicate the following costs:

1. $9.82.	2. $ .09.	3. $ .25.	4. $ 1.64.
5. $1.00.	6. $6.20.	7. $ .27.	8. $ 7.47.
9. $2.45.	10. $ .55.	11. $1.33.	12. $ .39.
13. $8.23.	14. $5.00.	15. $1.17.	16. $12.47.
17. $6.62.	18. $2.24.	19. $ .36.	20. $ 1.44.

Many articles are bought by the dozen and sold by the piece. In marking goods bought in this way it is necessary to divide the cost by 12. This division will be facilitated if the decimal equivalents of the 12ths, from $\frac{1}{12}$ to $\frac{11}{12}$ inclusive, are memorized.

Make such a table of equivalents and memorize it. Thus,

$\frac{1}{12} = .08\frac{1}{3}$; $\frac{7}{12} = .58\frac{1}{3}$.

Example. What is the cost of one hat at the rate of $29 a dozen?

SOLUTION. $\frac{1}{12}$ of $29 = $2\frac{5}{12} = $2.41\frac{2}{3}$.

Oral Work

What is the cost per article when the cost per dozen is

1. $27?	2. $34?	3. $64?	4. $11?	5. $ 3.40?
6. $14.30?	7. $7.50?	8. $19.50?	9. $46.60?	10. $37.40?

142. Showing Cost and Selling Price. When both the cost price and the selling price are shown, it is customary to write the cost price above and the selling price below a line, thus:

Cost of an article, $6.25.
Selling price, $8.00.
Key word, Fitzaubrey.

Price mark, $\frac{\text{uia}}{8.00}$.

Key words or phrases are sometimes used to mark the selling price as well as the cost, thus:

Cost key, "purchased it"
123456789 0 "t" is the repeater.

Selling key, "studying her"
12345678 90 "r" is the repeater.

Cost, $3.14.
Selling price, $4.25.

Mark, $\frac{\text{rpc}}{\text{dty}}$.

Written Work

Using the above keys, show markings for the following:

	Cost	Selling Price
1.	$1.15	$1.50
2.	$ .65	$ .90
3.	$1.30	$1.75
4.	$2.18	$2.50
5.	$1.25	$2.60

What per cent of profit would be realized from selling goods marked as follows, using "purchased it" as the cost key and "studying her" as the selling key?

6. $\frac{\text{phi}}{1.75}$. 7. $\frac{\text{ah}}{\text{ge}}$. 8. $\frac{\text{dh}}{\text{sty}}$. 9. $\frac{\text{at}}{\text{ny}}$.

Mark the cost of each of the following articles, using "Now Pay Quick" as the purchase key.

10. $3.00 per dozen.
11. $185.00 per hundred.
12. $11.52 per gross.
13. $8.64 per case of 4 doz.

Mark both the cost and the selling price of the following. Devise a cost key of your own, and mark the selling price by using the key "importance."

14. Cost $2.60, marked to gain 22%.

15. Cost $1.75, marked to gain 18%.

16. Cost $25.00, marked to gain 20%.

The following is a list of key words and phrases:

Buy for Cash	Our Last Key	Equinoctial	Now Be Quick
The Big Four	Republican	No Suit Case	You Mark His
Black Horse	Charleston	Frank Smith	Now Be Sharp
Buckingham	Bridgeport	Big Factory	He Saw It Run
Authorizes	Cumberland	Don't Be Lazy	Hard Moneys
United Cars	Dozen Black	Market Sign	What Prices

143. Adding the Buying Expenses to the Cost of the Goods. Since the buying expenses are considered a part of the cost of the goods purchased, many merchants add a portion of the buying expenses in marking the cost of each article. Records of total purchases and buying expenses per year are kept for several years, and the average per cent of buying expenses is determined, as shown by the following illustration: To determine a rate per cent of buying expenses for purchases made in 1915.

1912, Buying Expenses,	$ 785.75	Purchases,	$16,240.00
1913, Buying Expenses,	835.50	Purchases,	19,360.00
1914, Buying Expenses,	923.90	Purchases,	21,365.00
Total Buying Expenses,	$2545.15	Purchases,	$56,965.00

$2545.15 ÷ $56,965.00 = 4.46+%, the per cent of buying expenses.

In marking costs, 4.4% of the cost of each article should be added as buying expenses.

NOTE. Probably 5% might be used for convenience.

Written Work

1. Compute the per cent of buying expenses to be added to the wholesale cost of goods purchased in 1916. Carry the result to the nearest whole per cent.

Year	Purchases	Buying Expenses
1913	$10,246.80	$534.50
1914	12,726.95	592.90
1915	14,825.75	615.45

2. Find the marked cost of each of the following articles after adding the buying expenses. Use the per cent of buying expenses found in Problem 1.

a. Wholesale cost, $16.20.
b. Wholesale cost, 15.75.
c. Wholesale cost, 2.35.

3. At what price should each of these articles be sold to gain 23% on the total cost including buying expenses?

4. The purchases made by a store in 1915 were $3287.20; the buying expenses for the same year were $327.90. On the basis of these figures, what per cent should be added for buying expenses on purchases made in 1916? Approximate result to the nearest per cent.

5. What objection do you see to determining a per cent of buying expenses from the data of only one preceding year?

6. Find the marked cost of each of the following purchases, after adding the per cent of buying expenses. Use the per cent of buying expenses found in Problem 4.

a. Wholesale cost, $ 2.45.
b. Wholesale cost, 1.60.
c. Wholesale cost, 34.50.

7. At what selling price should each of the articles in Problem 6 be marked to gain $16\frac{2}{3}$% of the total cost?

CHAPTER XVI

PAYING FOR GOODS

144. Making Change. The payment of debts between persons in the same community is usually made with either cash or checks. When cash is the medium of payment, it is often necessary to "make change."

Speed and accuracy in making change are very desirable. The following method is generally used by experienced tellers and cashiers. Beginning with the amount of the purchase, take from the cash drawer enough small coins to bring the total to even dollars, using as few coins as possible, then take out dollars or larger denominations until the total equals the payment made.

Example. A five-dollar bill was given in payment for a purchase of \$.39. How should change be made?

SOLUTION. Take 1 penny,
1 dime,
1 fifty-cent piece,
4 dollars.

As a check on the accuracy of the change, say as you give the customer the money: "39 cents, 40, 50, \$1, \$5.00."

Oral Work

Following the method above, state what coins and bills should be given to make change for the following purchases, using the largest denominations possible:

	PURCHASE	PAYMENT		PURCHASE	PAYMENT
1.	\$ .07	\$ .50	2.	\$ .21	\$ .50
3.	\$ .56	\$ 1.00	4.	\$ .63	\$ 2.00
5.	\$1.36	\$ 5.00	6.	\$2.79	\$ 5.00
7.	\$3.66	\$ 5.00	8.	\$4.24	\$10.00
9.	\$5.70	\$10.00	10.	\$6.32	\$20.00
11.	\$ .98	\$ 5.00	12.	\$2.77	\$10.00

	Purchase	Payment		Purchase	Payment
13.	$ 3.46	$20.00	14.	$11.87	$15.00
15.	$14.42	$20.00	16.	$ 7.37	$ 8.00
17.	$11.13	$20.00	18.	$ 1.13	$20.00

145. Payments by Check. The payment of all bills by the actual transfer of money would be so inconvenient that the giving of checks has been substituted, and it is said that about 90% of all bills are now paid by checks.

146. Deposits and Withdrawals. Business men keep the greater part of their cash funds on deposit in banks or trust companies. When money is deposited, a deposit slip similar to the illustration below is filled out by the depositor, showing his name, the date, the item, and the amount of the deposit.

DEPOSITED BY

C. L. Stevenson

IN THE

STATE BANK OF OAK PARK

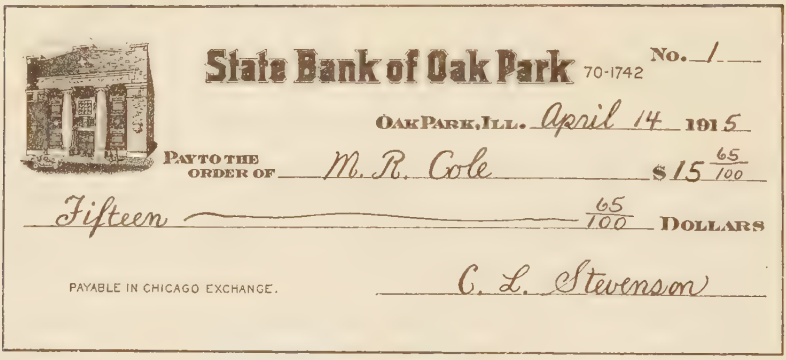
OAK PARK, ILL.

April 10 1915

	DOLLARS	CENTS
GOLD	40	00
SILVER	7	35
BILLS	14	00
Total	61	35

A Deposit Slip

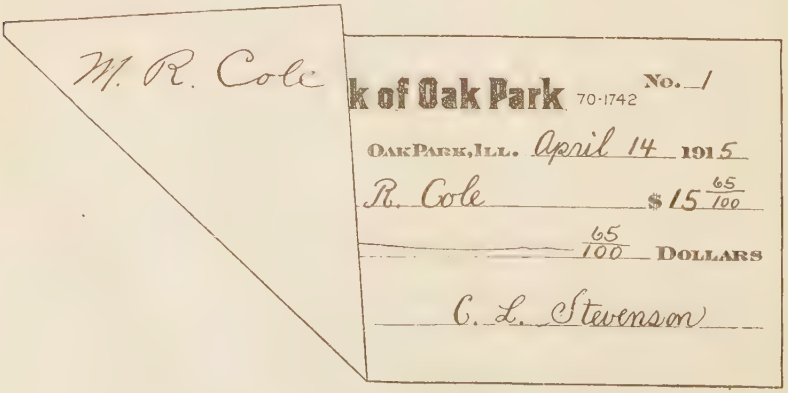
The depositor, wishing to pay a bill, draws a check ordering the bank to pay from the funds on deposit the sum of money stated on the check to the person named thereon. If the depositor wishes to draw cash from his account, he may make the check payable to "Self" or to "Cash."


State Bank of Oak Park 70-1742
No. 1
Oak Park, Ill. April 14 1915
Pay to the order of M. R. Cole $15 65/100
Fifteen 65/100 Dollars
Payable in Chicago Exchange.
C. L. Stevenson

A Check

C. L. Stevenson, the *drawer* of this check, is paying M. R. Cole, the *payee*, $15.65.

Depositors are credited by the bank with their deposits, and are charged with the checks drawn by them. Checks received from other people may be cashed at the bank, or they may be deposited the same as cash. All checks cashed or deposited must be indorsed; that is, they must be signed on the *back* by the person cashing or depositing them. Checks must also be indorsed when they are transferred to another person before being cashed at the bank. Indorsement should be made on the back of the left end of the check.



A Check Indorsed

M. R. Cole's indorsement before cashing the check.

INDORSEMENTS

147. Blank Indorsement. This form of indorsement is illustrated on the back of the check on page 174. It makes the check payable to bearer.

148. Indorsement in Full. This form is illustrated below.

Pay to the order of
I. B. Banning
M. R. Cole

Checks indorsed in full must be indorsed by the new owner before being cashed; if lost, there is less danger of their being cashed by a dishonest finder than if they were indorsed in blank.

If a check, note, or draft is not paid by the party primarily liable each indorser in turn becomes responsible for its payment. An indorser may limit his responsibility by writing the words "without recourse" above his signature. When this is done, the indorsement is said to be *qualified*.

New Netherland Bank
OF NEW YORK
DEPOSITED BY
V. E. Finch
NEW YORK. May 6, 1915

PLEASE LIST EACH CHECK SEPARATELY

	DOLLARS	CENTS
BILLS	16	00
GOLD	25	00
SILVER	12	45
CHECKS Buffalo	4	35
" Newark N.J.	14	23
" Boston	213	25
"		
"		
"		
"		
"		
"		
"		
Total	285	28

DEPOSIT SLIP SHOWING CASH AND CHECK DEPOSIT

Written Work

1. You are opening an account by depositing to-day in some bank of your city, gold, $5; silver, $2.25; bills, $7; and the three following

checks: 1st, drawn by Henry Bailey on the Merchants' National Bank of Fargo, North Dakota, amount $26.30; 2d, drawn by W. A. Owen on the Corn Exchange National Bank of Chicago, amount $8.35; 3d, drawn by Frank Mitchell on Carter's Bank, Eldora, Missouri, amount $12.40.

Rule a deposit slip and enter this deposit properly.

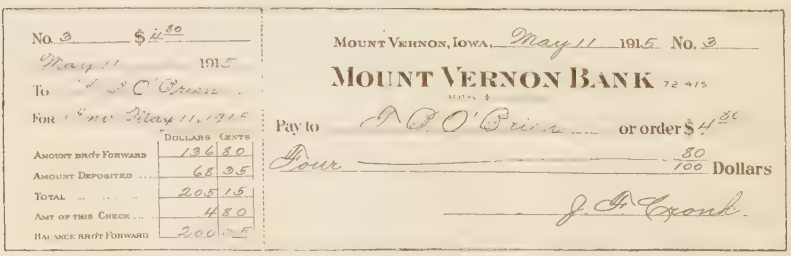
2. On a blank sheet of paper draw a check on the bank in which you have just made this deposit, paying F. G. Benton $23.25.

What is your balance in the bank after drawing this check?

3. Seven days ago you bought goods from F. G. Peterson, amounting to $28.50. The terms of the invoice were 2/10; N/30. To-day you pay the bill by check. Draw the check.

What is your balance in the bank?

149. The Check Book. Blank checks are furnished by the bank, bound together in a check book. Checks are usually attached to stubs, on which are recorded, at the time of drawing the check, its number, the date, the name of the payee, the purpose for which it was given, and the amount. On the stub there is also kept a record of the balance remaining in the bank. As shown by the following illustration, a balance of $136.80 was brought over from May 10, and on May 11 a deposit of $68.35 was made and a check for $4.80 was drawn.


No. 3 $4.80
May 11 1915
To P. O'Brien
For One May 11, 1915
DOLLARS CENTS
Amount brot Forward 136 80
Amount Deposited 68 35
Total 205 15
Amt of this Check 4 80
Balance brot Forward 200 35
Mount Vernon, Iowa, May 11 1915 No. 3
MOUNT VERNON BANK
Pay to P. O'Brien or order $4.80
Four 80/100 Dollars
J. A. Crank

Written Work

1. Using the following information, rule a check and stub similar to the preceding illustration, enter the necessary facts, and draw the check.

Name of bank, Fourth National, your city.

Balance from yesterday, $103.27. Deposit of $19.38.

Check No. 6, drawn by you to-day, payable to G. D. Fitzgerald and Co., in payment of the invoice purchased from them eight days ago. Amount of invoice, $73.28; terms, 2/10; N/60.

Write the check and stub.

What is the new balance?

150. The Bank's Accounts with its Depositors. All banks keep a depositors' ledger. Usually a page is devoted to the deposits and checks of each depositor.

Williams & Harris

DATE			DEPOSITS		CHECKS		BALANCE	
1915								
Jan.	6		143	65			143	65
	8				26	25	117	40
	12		68	60	13	25	172	75
	16				6	60		
					12	40		
					3	75	149	80
	21		6	60	43	74		
					102	73	9	33

An Account in a Depositors' Ledger

Written Work

Rule a page of a depositors' ledger, similar to the preceding illustration, and enter the following deposits and checks made by yourself, computing the daily balances. In computing balances, use the method of adding and subtracting explained in Section 13.

March 2, Deposit $127.80. March 3, Check $26.14. March 4, Check $23.19. March 5, Deposit $114.32; Check $12.19. March 6, Checks $13.14, $12.17. March 10, Deposits $32.76, $22.14; Check $17.14. March 11, Checks $26.19, $127.14. March 13, Deposits $12.19, $76.27; Checks $32.15, $27.19,

$13.81. March 16, Deposit $27.94. March 18, Checks $23.42, $27.93.

151. The Clearing House. Any bank will cash the checks drawn on other banks, provided the person presenting them is known at the bank or is properly identified. For example, if you are given a check on the First National Bank, you can cash it or deposit it at the Second National Bank. The Second National then collects the amount of the check from the First National.

When there are several banks in the same city, the checks are collected through a Clearing House. The representatives of the various banks meet at the Clearing House, bringing the checks which they have paid for other banks. Each bank is credited with the amount of the checks which it has paid for other banks, and debited with the amount of the checks which all other banks have paid for it. The Clearing House collects from those having debit balances and pays those having credit balances.

Let us suppose that there are three banks in a city: the First National, the First State, and the City National. The representatives will meet, each bringing the checks paid by his bank for the other banks the preceding day, and a sheet showing the amount paid for each of the other banks. The statements brought by the representatives of the banks are shown below:

FIRST NATIONAL

PAID FOR	AMOUNT
First State	$16,265.00
City National	12,146.80
Total	$28,411.80

FIRST STATE

PAID FOR	AMOUNT
First National	$ 8,368.28
City National	9,208.48
Total	$17,576.76

CITY NATIONAL

PAID FOR	AMOUNT
First National	$12,875.00
First State	4,738.80
Total	$17,613.80

At the Clearing House a sheet similar to the following is prepared. In the debit column is placed the total value of checks paid *for* each bank; in the credit column is placed the total value of checks paid *by* each bank. The balance columns show the balances payable by or to each bank.

MANAGER'S SHEET

BANK	DEBITS	CREDITS	DEBIT BALANCE	CREDIT BALANCE
First National	$21,243.28	$28,411.80		$7168.52
First State	21,003.80	17,576.76	$3427.04	
City National	21,355.28	17,613.80	3741.48	
Totals	$63,602.36	$63,602.36	$7168.52	$7168.52

The First State and the City National each turns over to the Clearing House the amount of its debit balance; the Clearing House pays this amount to the First National. In this way payment of over sixty-three thousand dollars is made by the actual transfer of only a little more than seven thousand dollars.

Written Work

Prepare a Manager's Sheet, and find the debit or credit balance of each bank in the following Clearing House Association. The amount of the checks paid by each bank is shown by the debit sheets which follow.

HARPER'S STATE BANK

PAID FOR	AMOUNT
Harper's State	
First National	$23,756.27
Traders'	14,259.95
Merchants' National	25,726.87
Farmers' & Mechanics	9,243.65
First State	12,060.26

First National Bank

Paid for	Amount
Harper's State	$13,263.37
First National	
Traders'	8,836.39
Merchants' National	14,856.74
Farmers' & Mechanics	8,345.93
First State	8,325.57

Traders' Bank

Paid for	Amount
Harper's State	$5,273.85
First National	12,573.82
Traders'	
Merchants' National	7,252.55
Farmers' & Mechanics	11,257.37
First State	10,456.25

Merchants' National

Paid for	Amount
Harper's State	$3,123.64
First National	6,027.30
Traders'	10,936.40
Merchants' National	
Farmers' & Mechanics	7,238.37
First State	9,235.35

Farmers' & Mechanics

Paid for	Amount
Harpers' State	$2,346.75
First National	5,236.73
Traders'	4,232.25
Merchants' National	7,727.86
Farmers' & Mechanics	
First State	2,005.39

FIRST STATE

PAID FOR	AMOUNT
Harper's State	$8,946.27
First National	1,856.23
Traders'	11,472.75
Merchants' National	923.88
Farmers' & Mechanics	2,423.73
First State	

Which banks will pay to the Clearing House, and how much will each pay?

Which banks will receive from the Clearing House, and how much will each receive?

152. Exchange on Checks. Checks are frequently used to send money to people in cities other than the one in which the drawer of the check resides. For example, if you have money deposited in the Home National Bank of your city, you may draw a check against this deposit and send it to Mr. Perkins of Omaha. Mr. Perkins will take it to his bank in Omaha, which will pay the check, even though it has no knowledge of you or your deposit in the Home National Bank. The fact that Mr. Perkins is known at his bank, and that he agrees to refund the money if the check proves worthless, is sufficient protection for his bank.

When banks pay out-of-town checks, they incur a certain amount of expense in collecting them. To cover this expense they often require persons for whom they cash checks to pay them "Exchange." When exchange is charged, a common rate is $\frac{1}{10}$ of one per cent of the face of the check, with a minimum charge of from 10 cents to 25 cents.

Example. Mr. Black, who lives in Bloomington, Illinois, received two checks in his mail. One was from Wm. Harris, of Lincoln, Nebraska, for $365.85; the other from H. B. Felter, of Cincinnati, Ohio, for $28.25. The bank at which Mr. Black cashed these checks charges for exchange $\frac{1}{10}$ % of the face of each

check, or a minimum of 10 cents. What was the exchange on the checks?

SOLUTION. $\frac{1}{10}\%$ of \$365.85 = \$.37.
$\frac{1}{10}\%$ of \$28.25 = \$.028.

Since \$.028 is below the minimum, 10 cents exchange is charged.
The total exchange is \$.47.

Written Work

Prepare deposit tickets for the following:

1. Deposited by Wm. Frend, of Peoria, Illinois, in the Peoria State Bank, on August 7, 1915:

Gold, \$25; silver, \$37.50; currency, \$17; and checks as follows:

Check drawn by D. P. Snyder of Amboy, Illinois, \$65.75.
Check drawn by Hammond & Davis of Lima, Iowa, \$236.57.
Check drawn by B. J. Harris of Springfield, Illinois, \$235.
The bank charges $\frac{1}{10}\%$ exchange. Minimum, 15 cents.
Find the total of the items and deduct the charge for collection.

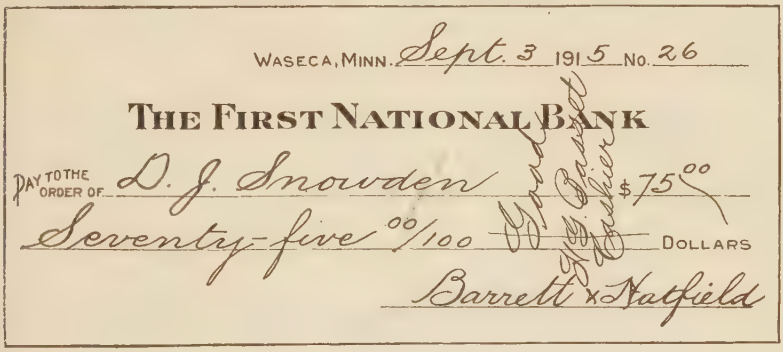
2. Deposited by G. D. Bernham of Hawkeye, Iowa, in the Hawkeye State Bank, on August 20, 1915.

Check drawn by S. Y. Benton of Clarinda, Iowa, \$75.88.
Check drawn by D. W. Galbreath of Morris, Illinois, \$57.93.
Check drawn by F. J. Ressler of Pasadena, California, \$346.49.
Check drawn by D. T. Bailey of Buffalo, New York, \$48.95.
The bank charges exchange on the California and New York checks; rate, $\frac{1}{10}\%$, minimum 25 cents each.

NOTE. It is the custom of some banks to cash checks on banks in near-by states without charging exchange.

153. Certified Checks. If you wish to assure the person to whom you send a check that you have sufficient money in the bank to pay the check, you may have the check certified. Draw the check in the usual way and take it to your bank. The cashier will write "Accepted" or "Certified" or "Good" across its face and will sign his name. He will then set aside from your deposit the amount of the check, holding it in the bank's funds until the check is presented for payment. Certifying the

check reduces the receiver's risk of cashing a worthless check, but it does not relieve him from a possible exchange fee.

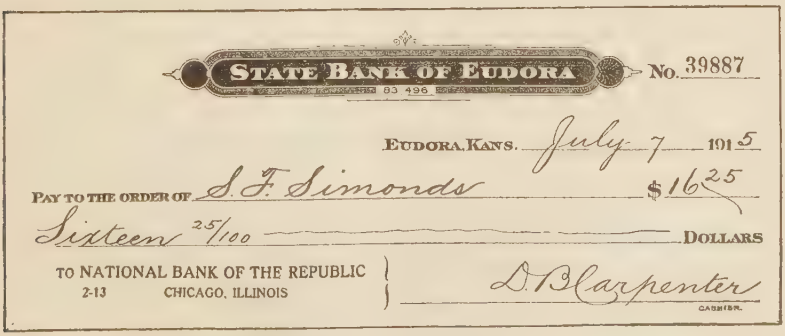

WASECA, MINN. Sept. 3 1915 No. 26
THE FIRST NATIONAL BANK
PAY TO THE ORDER OF D. J. Snowden $75.00
Seventy-five 00/100 DOLLARS
Barrett & Hatfield

A Certified Check

154. Bank Drafts. Banks keep funds on deposit in the banks of the larger cities, and draw checks on these city banks just as you may draw a check on the bank in which you have deposited money. An order drawn by one bank against its deposit in another bank is called a "Bank Draft."

155. Advantages of Bank Drafts. A bank draft is preferable to a check when sending money from one town to another, for two reasons :

a. The draft is drawn by a bank, while the check is drawn by an individual. There is therefore a greater certainty that the bank draft is genuine, and will be paid when presented.


STATE BANK OF EUDORA
No. 39887
EUDORA, KANS. July 7 1915
PAY TO THE ORDER OF S. F. Simonds $16.25
Sixteen 25/100 DOLLARS
TO NATIONAL BANK OF THE REPUBLIC
2-13 CHICAGO, ILLINOIS
D. B. Carpenter
CASHIER.

A Bank Draft

b. Bank drafts are drawn on banks in large cities, while checks may be drawn on banks in small towns. There is less expense, therefore, incurred in collecting a bank draft than in collecting a check, and banks seldom charge exchange when cashing bank drafts.

EXPLANATION. D. B. Carpenter is the cashier of the State Bank of Eudora. This bank has funds on deposit in the National Bank of the Republic, Chicago. When the National Bank of the Republic pays this draft, it will deduct the amount from the balance of the Eudora Bank.

S. F. Simonds lived in Eudora and wished to send $16.25 to F. E. Craig of Bloomington, Illinois. When he purchased the draft from his bank, he had it made payable to himself. He might have had it made payable to F. E. Craig, but if he had done so, his (Simonds's) name would not have appeared on the draft, and if it had become separated from the letter accompanying it, Craig might have had difficulty in telling from whom he received it.

After receiving the draft, Simonds indorsed it as follows:

Pay to the order of
F. E. Craig.
S. F. Simonds.

and sent it to Mr. Craig.

Mr. Craig indorsed it and cashed it at the Corn Belt National Bank of Bloomington.

We will now follow the steps by which the Corn Belt National collected the draft. This bank deposits funds with the Continental National Bank of Chicago, which is called its "correspondent," and it therefore sent this draft, with others paid the same day, to the Continental National bank as a deposit.

The Continental National sent the draft to the Clearing House, which collected it from the National Bank of the Republic.

After paying the draft, the National Bank of the Republic charged the amount to the State Bank of Eudora.

The preceding statement may be summarized as follows:

State Bank of Eudora sells the draft to

↓

S. F. Simonds, who paid for it either with cash or a check; and sent it to

↓

F. E. Craig, who gave Simonds credit for $16.25; and deposited the draft at

↓

Bloomington Bank, which gave Craig $16.25; and sent the draft to

↓

Continental National, which gave the Bloomington Bank credit for $16.25; and collected the draft through the Clearing House from

↓

National Bank of the Republic, which paid the Continental National $16.25, and charged Eudora Bank $16.25.

INDORSEMENTS ON THE MODEL BANK DRAFT

Pay to the order of
F. E. Craig
S. F. Simonds

Pay to the order of the
Corn Belt National Bank
F. E. Craig

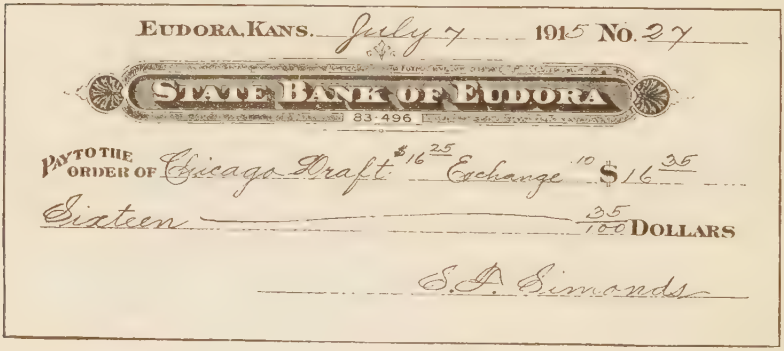
Pay to the order of
Continental National Bank.
All prior indorsements guaranteed.
CORN BELT NATIONAL BANK,
BLOOMINGTON, ILL.,
F. A. FREND, Cashier.

Pay to the order of
Any Bank, Banker, or Trust Co.
Prior indorsements guaranteed.
July 12, 1915.
CONTINENTAL NATIONAL BANK.
M. B. JONES, Cashier.

These indorsements indicate the successive owners of the draft. The last two indorsements are made with rubber stamps. The last one is made payable to "any Bank, Banker, or Trust Company," for convenience in collecting through the Clearing House.

156. The Cost of Bank Drafts. When a bank sells a draft, it gives the purchaser the benefit of its deposit relations with a large city bank. For this convenience it may make a charge called "Exchange." It will be noted that exchange on a check is paid by the person receiving and cashing it, while the exchange on a draft is paid by the person purchasing and sending it. When exchange is charged, $\frac{1}{10}$ % with a minimum of 10 cents is a customary fee.

The following is a copy of the check given by S. F. Simonds to the State Bank of Eudora to pay for the draft purchased. There was a charge of 10 cents for exchange.


EUDORA, KANS. July 7 1915 No. 27

STATE BANK OF EUDORA

83-496

PAY TO THE ORDER OF Chicago Draft $16.25 Exchange .10 $16.35

Sixteen 35/100 DOLLARS

S. F. Simonds

Written Work

1. You owe H. J. Palmer, of Toledo, Ohio, $27.35. You buy a draft, payable to yourself for this amount, from the Merchants' State Bank of your city, of which A. R. Burton is the cashier. The draft is drawn on the Home National Bank of New York City. Exchange, 10 cents. Draft is made payable to you.

Write a draft similar to the one which the bank would give you, and the check which you would give the bank in payment.

Indorse the draft. How soon can Mr. Palmer get his money?

2. Mr. Palmer cashed the draft at the High Street National Bank of Toledo, B. F. Ohren, Cashier. The High Street National sent it to the Bankers' National Bank of New York, G. B. Martin, Cashier. The Bankers' National collected it through the Clearing House. Show the indorsements on the draft.

3. Find the cost of a draft sent to pay an invoice of $246.50, less a cash discount of 2%. Exchange, $\frac{1}{10}$% of the face of the draft; minimum, 15 cents.

4. On October 8, you purchased from Benedict & Meredith, Pittsburgh, Pennsylvania, an invoice amounting to $794.86. Terms, 2/15; N/60. On October 16, you paid the invoice by a draft purchased at the Claim Street State Bank of your city, J. D. Haines, Cashier. The draft was drawn on the First National Bank of New York. Exchange, $\frac{1}{10}$%.

Write the draft which you received, the indorsement, and the check you gave to pay for the draft.

157. Postal Money Orders. A postal money order is an order drawn by one postmaster on the postmaster at some other office, calling for the payment of a stated sum of money to the person named on the order. Postal money orders are commonly used to make payments by mail. They are issued in any amount from $.01 to $100.00. As a means of protection, no order is issued for more than $100.00. If it is desired to send more than this amount, additional orders may be purchased.

Fees for money orders payable in the United States (which includes Hawaii and Porto Rico) and its possessions, comprising the Canal Zone, Guam, the Philippines, as well as in Bermuda, British Guiana, British Honduras, Canada, Cuba, Mexico, Newfoundland, at the United States Postal Agency in Shanghai (China), in the Bahama Islands, and in certain other islands in the West Indies, are as follows:

For orders from	$ 0.01	to $ 2.50	. . . .	3 cents
from	2.51	to 5.00	. . . .	5 cents
from	5.01	to 10.00	. . . .	8 cents
from	10.01	to 20.00	. . . .	10 cents
from	20.01	to 30.00	. . . .	12 cents
from	30.01	to 40.00	. . . .	15 cents
from	40.01	to 50.00	. . . .	18 cents
from	50.01	to 60.00	. . . .	20 cents
from	60.01	to 75.00	. . . .	25 cents
from	75.01	to 100.00	. . . .	30 cents

Examples. 1. What is the cost of an order for $27.35?

SOLUTION. $27.35 lies between $20.01 and $30.00, and the rate is 12 cents.
Total cost of order, $27.35 + .12 = $27.47.

2. What is the cost of sending $267.95 by postal money orders?

SOLUTION. Two orders for $100.00 each, and one order for $67.95, will be purchased.

Fees of two $100.00 orders, 60 cents.
Fee of $67.95 order, 25 cents.
Total fees, 85 cents.
Total cost of orders, $267.95 + .85 = $268.80.

158. Bank Drafts and Postal Money Orders Contrasted. Note the following differences between a bank draft and a postal money order:

A bank draft may be indorsed as many times as desired.

A postal money order may be indorsed only once.

A bank draft may be cashed at *any* bank.

A postal money order must be presented to the post office on which it is drawn, or to a bank which can cash it at that post office.

A bank draft is payable as soon as it is presented to a bank.

At the time of issuing a money order the issuing postmaster sends a notice to the paying postmaster. A postal money order will not be paid until the paying office has received this notice. However, this does not usually cause any delay.

159. Express Money Orders. Express money orders are similar in many respects to postal money orders. They can be purchased from the agent of the express company, and are payable from the funds of the express company on deposit in various banks specified in the order.

No order is issued for an amount larger than $50. If it is desired to send a larger amount, additional orders must be purchased.

Express Money Order Rates. The rates for express money orders are the same as for postal money orders, although no order is issued for more than $50.

Example. What will be the total fee for the transfer of $241.75 by express money orders?

SOLUTION. Four $50 orders at the rate of 30 cents per hundred will cost 60 cents.

One order for $41.75 will cost 18 cents. Total, 78 cents.

160. Telegraph Money Transfers. Money may be transferred by telegraph when there is an urgent necessity for immediate payment. The rates for this service between points in the United States are determined as follows:

To the tolls for a fifteen-word message between the office of deposit and the office of payment add the following charges:

For $25.00 or less	25 cents
25.01 up to $50	35 cents
50.01 up to 75	60 cents
75.01 up to 100	85 cents

For amounts above $100 add (to the $100 rate) 25 cents per hundred (or any part of $100) up to $3000.

For amounts above $3000 add (to the $3000 rate) 20 cents per hundred (or any part of $100).

Examples. 1. What is the charge for sending $25 or less to a point where the fifteen-word rate is 65 cents?

SOLUTION.

$ .25	Minimum charge
.65	Tolls on 15-word message
$ .90	Total charge

2. What is the total charge for transfer of $105 if the tolls on a fifteen-word message are 65 cents?

SOLUTION.

$ .85	Charge for first $100
.25	Charge for fraction of second $100
.65	Tolls on 15-word message
$1.75	Total rate

Written Work

1. Assuming that you owe the following bills:

F. G. Young, $39.40;
P. S. Sanborn, $112.75;
H. L. Colwell, $416.25, less 3%;

determine the fees for the purchase of either postal or express money orders for transferring money to make payments. Complete the following blank form.

Name	Net Amount of Bills	Fees for Money Orders	Total Cost	Denominations of Orders Received from Ex. Co.	Denominations of Orders Received from P. O.

2. What is the charge for a telegraph money transfer of $23.15 to a point where the fifteen-word message rate is 85 cents?

3. What is the charge for a telegraph money transfer of $52.85 to a point where the fifteen-word message rate is $1.30?

4. What is the total cost of a telegraph money transfer for $215 to a point where the fifteen-word rate is 65 cents?

CHAPTER XVII

COLLECTING BILLS

161. Statements. At periodical intervals, usually on the first day of each month, merchants send a statement to their customers.

STATEMENT

ACCOUNT NO. 5422

WEBSTER & McCLELLAN

16 JEFFERSON STREET

F. B. Turner,
La Grange, Ill.

CHICAGO, May 1, 1915

All bills are due the first of the month after purchase.
This statement is intended to show you the condition of your account on our books. If this statement does not correspond with your accounts please notify us. If correct, please remit.

	To balance, as per former statement	17.65	
	To Mdse, as per invoice		
April 3		23.78	
April 15		19.65	
April 23		27.84	88.92
	Credits		
April 20	By Mdse returned	2.85	
April 26	By Cash	75.00	77.85
	Balance		11.07

This statement shows the date and amount of each purchase, the date and amount of each payment, and the balance due. The items sold are not enumerated in the above statement because the invoices may be consulted to obtain this information.

HENDERSON & BISHOP
143 WEST MONROE AVENUE

R. F. Bailey,
West Union, Iowa.

CHICAGO, July 1, 19 15

Interest charged on overdue accounts.
All claims for correction must be made on receipt of statement.

DATE	ITEM	DEBIT	CREDIT	BALANCE
June 1	Balance, May 31			75.60
June 2	Cash		75.00	.60
June 5	Mdse	34.55		35.15
June 11	Mdse	17.26		52.41
June 15	Returned goods		4.35	48.06
June 24	Mdse	5.96		54.02

ANOTHER FORM OF STATEMENT

Written Work

1. Rule a statement similar to the first illustration, and enter the following: Your transactions with S. J. Smith, Fayette, Missouri.

Balance, August 1, $127.86.

Purchases, August 3, $72.85; August 11, $23.89; August 14, $75.23; August 20, $14.76; August 28, $38.97.

Payments, August 2, Cash $126.00; August 13, Returned merchandise, $15.00.

Statement rendered September 1.

2. Rule a statement similar to the second illustration, and enter the following transactions with Beardsley & Russel, Montgomery, Alabama.

Balance, October 1, 1915, $32.11.

Purchases, October 5, $45.89; October 9, $23.87; October 12, $243.87; October 15, $65.00; October 22, $52.57; October 26, $40.50; October 29, $50.25.

Payments, October 8, $50.00; October 22, $125.00; October 24, $40.00.

Returned merchandise, October 20, $5.50.

Statement rendered November 1.

162. Commercial Drafts. Commercial drafts offer an effective method of collecting accounts. A commercial draft is an order drawn by the party to whom money is due, requesting the debtor to pay a stated sum of money either to the drawer of the draft or to a third party mentioned in the draft.

Two and Three Party Drafts. If a draft requests the debtor to pay money to the drawer, it is called a **two-party draft.** Such drafts are collected through a bank.

If a draft requests the debtor to pay money to a third party, it is called a **three-party draft.**

Sight and Time Drafts. If a draft requests immediate payment, it is called a **sight draft.** Such a draft is payable at once, without acceptance.

If a draft is to be paid after a stated time, it is called a **time draft.**

163. Parties to a Draft. The **drawer** is the person who draws the draft. The **drawee** is the person on whom the draft is drawn, and who is requested to pay the money. The **payee** is the person to whom the money is to be paid. In case of a two-party draft, the drawer is also the payee.

164. Reasons for Drawing Drafts.

(*a*) To effect prompt payment of invoices.

1. How a sight draft is used: Wholesale houses often make terms similar to the following: "Sight draft in ten days, less 2 %." At the expiration of ten days from the date of sale, the selling merchant sends a sight draft for the net amount of the bill, to the purchasing merchant's bank. The bank presents it to the purchaser for collection.

2. How a time draft is used: If the terms of the sale were "Thirty-day draft," the selling merchant would, at the time of

making the sale, draw a thirty-day draft on the purchaser and send it to him for acceptance. After the draft has been accepted, it has the same value as a note because the purchaser has agreed to pay it when due. The seller may borrow money from a bank, giving the accepted draft as security.

(*b*) As a means of collection. When a bill is overdue, and the debtor is slow in making settlement, a draft, to be collected by his bank, will often bring about a settlement.

(*c*) To make C. O. D. shipments by freight. When a merchant sells goods to be shipped by freight, he receives a bill of lading from the railroad company. This bill of lading is the railroad's receipt for the goods. To make a C. O. D. shipment the seller obtains an Order Bill of Lading from the railroad. The purchaser cannot get the goods from the railroad without surrendering the bill of lading. Therefore, a selling merchant, instead of sending the bill of lading direct to the purchaser, attaches it to a sight draft and sends the draft and the bill of lading to a bank in the purchaser's town.

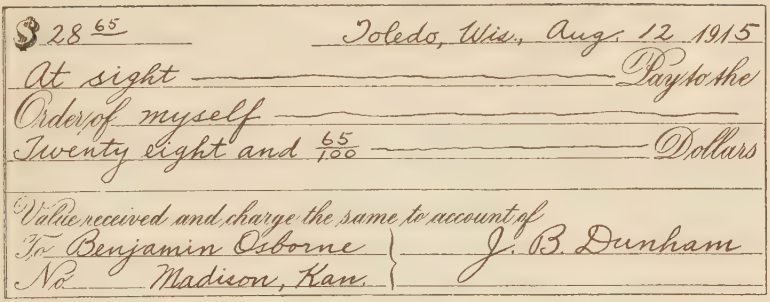
In order to get the goods the purchaser must have the bill of lading; in order to get the bill of lading he must pay the draft at the bank.

(*d*) To avoid the transfer of money. If

Jones owes Smith, and
Smith owes Brown,

Smith may collect the debt from Jones and pay his own debt to Brown, or he may request Jones to send the money direct to Brown. If Jones and Brown live in the same town, both debts can be paid without sending the funds through the mail, even though Smith lives in a distant city. A three-party draft is used for this purpose, Smith requesting Jones to pay Brown.

165. Two-party Sight Draft. Benjamin Osborne of Madison, Kansas, owes J. B. Dunham of Toledo, Wisconsin, $28.65. In order to collect this bill, Mr. Dunham draws the following draft on Mr. Osborne.


$28 65/100 Toledo, Wis., Aug. 12 1915
At sight Pay to the
Order of myself
Twenty Eight and 65/100 Dollars
NO PROTEST TAKE THIS OUT BEFORE PRESENTING.
Value received and charge the same to account of
To Benjamin Osborne
No Madison, Kan.
J. B. Dunham

Mr. Dunham then indorses the draft as follows:

Pay to the order of
The First State Bank of Toledo
for collection only.
J. B. Dunham.

and gives it to his bank to collect.

The bank, in turn, indorses it as follows:

Pay to the order of
The Farmers National Bank of Madison
for collection only.
First State Bank,
Toledo, Wisconsin,
E. J. Baxter, Cashier.

and forwards it to the bank at Madison to collect. The Madison bank presents it to Mr. Osborne for payment. Mr. Osborne is not obliged to pay this draft, but if he owes the bill, he will probably do so in order to keep his standing good at his bank.

If Osborne pays the draft, the bank stamps a receipt thereon, and gives the draft to Osborne. The bank keeps a certain amount as a collection fee, and returns the balance to the bank at Toledo, where Mr. Dunham receives the proceeds in cash or as a deposit credit.

If Osborne does not pay the draft, it is returned to the bank at Toledo with a statement of the reason for non-payment.

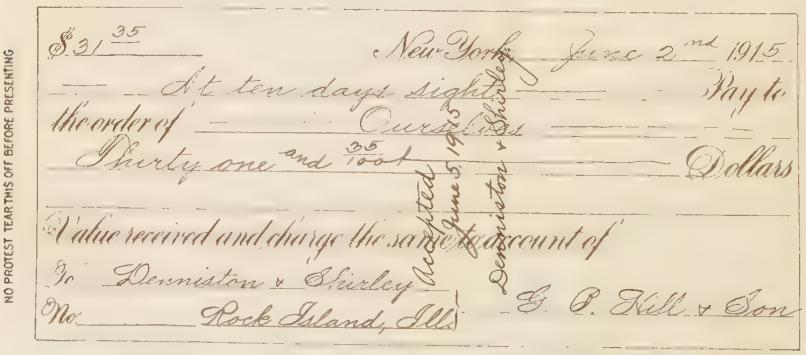
166. Rates for Collecting Drafts. "Collection" rates, like exchange rates, differ. A common rate is $\frac{1}{10}$ %, with a fixed minimum.

167. Two-party Time Draft. By substituting for the words "At sight" in the preceding illustration, such words as

Ten days after date,
At thirty days' sight, or
Thirty days after sight

the draft becomes a time draft. Instead of paying this draft when it is first presented, Mr. Osborne will accept it; that is, he will agree to pay it when it is due. Acceptance consists of writing across the face of the draft the word "Accepted" and signature.

168. Maturity of Time Drafts. Drafts payable "after sight" or "at — days' sight" are payable a stated number of days after acceptance, and the acceptance should therefore be dated. Drafts payable "after date" become due a stated number of days after the date of the draft. The acceptance, therefore, does not need to be dated, but it usually is.


NO PROTEST TEAR THIS OFF BEFORE PRESENTING

$31.35 New York June 2nd 1915

At ten days sight Pay to

the order of Current Date

Thirty one and 35/100 Dollars

Value received and charge the same to account of

To Denniston & Shirley

No. Rock Island, Ills. G. P. Hill & Son

Accepted June 5, 1915 Denniston & Shirley

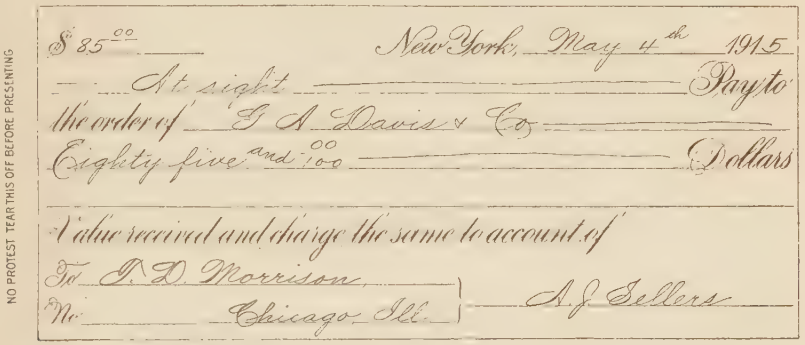
An Accepted Time Draft

This draft is due ten days after June 5, or June 15.

169. Three-party Drafts. T. D. Morrison of Chicago owes A. J. Sellers of New York $85.00.

Mr. Sellers owes G. A. Davis & Co. of Chicago $150.00.

Mr. Sellers draws the following draft, asking Mr. Morrison to pay G. A. Davis & Co. $85.00.


NO PROTEST TEAR THIS OFF BEFORE PRESENTING
$85.00
New York, May 4th 1915
At sight
Pay to
the order of G. A. Davis & Co.
Eighty five and 00/100
Dollars
Value received and charge the same to account of
To D. D. Morrison,
No. Chicago, Ill.
A. J. Sellers

THREE-PARTY SIGHT DRAFT

Oral Work

What two parties make a payment when this draft is used?

Explain how one transfer of money is saved?

How much will Sellers owe G. A. Davis & Co. after they have received this draft?

Three-party drafts may also be made payable after a stated time by using the terms explained on page 196.

Written Work

1. On July 13, you purchased from A. B. Butler of Minneapolis, Minnesota, an invoice of goods amounting to $123.60; terms, sight draft in 10 days, less 3%. On July 20, Mr. Butler drew a sight draft on you for the net amount of the bill. He indorsed the draft, and gave it to the Fourth Street Bank of Minneapolis for collection. The Minneapolis bank indorsed it and sent it to the First National Bank of your city. The draft was presented to you by the bank on the 23d of July.

Write the draft with indorsements.

If the First National Bank charged 15 cents for collection, how much did Mr. Butler receive in payment of the draft?

How much did you pay to the bank?

2. You purchased an invoice of goods from Winthrop and Monroe of Toledo, Ohio. Amount of invoice, $365.00. It was your first transaction with them and consequently you were unknown to them. As you wanted the goods immediately and did not wish to delay while inquiries were made regarding your credit, you instructed them to send the goods by freight and send a sight draft, with bill of lading attached, to the First National Bank of your city. The sight draft, which was dated yesterday, arrived this morning. It was made payable to Winthrop and Monroe and was indorsed to the order of the bank.

Write the draft and show the indorsement made by Winthrop and Monroe.

Write a check in favor of the bank to pay the draft.

What will you receive in exchange for the check?

How will Winthrop and Monroe get their money?

3. On September 3, you purchased an invoice of goods from S. L. Norton of Springfield, Illinois. Terms, thirty-day draft from date of sale, less 1%. Amount of invoice, $75.00. On September 5, you received the draft in the mail; it was dated September 3, and was payable 30 days after date. You immediately accepted the draft and returned it to Mr. Norton.

Write the draft, and show your acceptance.

When will you be expected to pay the draft?

4. You owed S. D. Briggs of Winona, Minnesota, $75.00; Mr. Briggs owed F. R. Tuttle of your city, $136.00. On June 19, Mr. Briggs drew a sight draft on you for the amount of your debt, and sent it to Mr. Tuttle as part payment of his debt. Mr. Tuttle presented it to you. You gave Mr. Tuttle cash in payment of the draft, and received the draft as a receipt.

Write the draft.

How much did Briggs owe Tuttle after the draft was paid?

CHAPTER XVIII

ACCOUNTS

An account is an orderly record of the transactions pertaining to any one person or thing.

170. The Cash Account. In keeping a record of cash,

All cash receipts are entered on the left, or debit side;

All cash payments are entered on the right, or credit side;

The difference between the total receipts and total payments is the balance of cash on hand.

The following transactions are recorded in the model cash account in the illustration.

April 6, 1915, C. D. Smith invested $5000 in business; April 7, he paid $1640 for goods; April 9, he paid his store rent, $75; April 10, he received $275 for merchandise; April 13, he received $36 from John Appleton; April 14, he paid F. G. Barton $128; April 16, William Hobart paid him $60 on account; April 28, he paid D. F. Hilton $30 on account.

Cash

1915				1915			
April	6	C. D. Smith	5000—	April	7	Mdse. Purchase	1640—
	10	Mdse. Sale	275—		9	Store Rent	75—
	13	John Appleton	36—		14	F. G. Barton	128—
	16	William Hobart	60—		28	D. F. Hilton	30—
					30	Balance	3498—
			5371—				5371—
May	1	Balance	3498—				

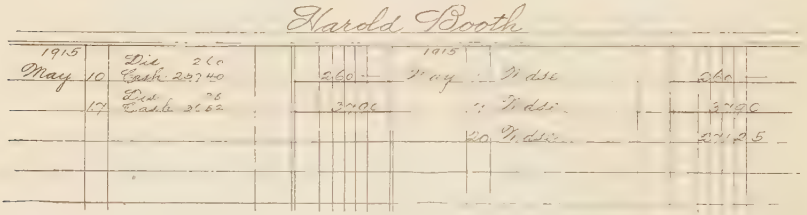
171. Personal Accounts. Transactions with persons are recorded in personal accounts. Persons to whom goods are sold are debited for all goods sold to them on account; they are credited for payments made by them on account.

Persons from whom goods are bought, are credited for goods purchased from them on account; they are debited for payments made to them on account.

The two illustrations which follow will show the two common rulings for personal accounts.

May 1, 1915, purchased from Harold Booth, merchandise, $260.

May 7, purchased merchandise from him, $37.90; May 10, paid him for the invoice of May 1, less 1% discount, Cash $257.40, Discount $2.60; May 17, paid him for the invoice of May 7, less 1% discount, Cash $36.52, Discount $.38; May 20, purchased merchandise, $271.25.



June 1, 1915, sold to B. A. Newcomber, goods amounting to $345.70; June 4, sold to Newcomber merchandise amounting to $38.45; June 11, received payment for invoice of June 1, less 2%, Cash $338.79, Discount $6.91; June 12, sold Newcomber an invoice amounting to $83.92; June 14, received payment for the invoice of the 4th, less 1%, Cash $38.07, Discount $.38.

B A Newcomber

Date		Sale	Cash	Discount	Balance
1915					
June	1	345 70			345 70
"	4	38 45			384 15
"	11		338 79	6 91	38 45
"	12	83 92			122 37
"	14		38 07	38	83 92

172. Accounts Receivable and Accounts Payable. If a person's account is larger on the debit side, it shows a balance owed *by* that person; it is therefore an account receivable, and the balance is a resource.

If a person's account is larger on the credit side, it shows a balance owed *to* that person; it is therefore an account payable, and the balance is a liability.

Written Work

1. Rule a cash account and enter the following transactions; find the balance, and rule the account.

August 2, you invest $5000 in a grocery business; August 3, purchase merchandise, paying cash for the same, $1345.75; August 6, receive cash for merchandise sold, $127.50; August 10, receive cash for goods sold, $50.25; August 12, pay for advertising, $5.60; August 17, receive cash for merchandise sold, $23.74; August 18, pay cash for an invoice of merchandise, $56.35; August 21, receive cash from Henry Belmont on account, $45.80; August 23, pay cash to Oscar Haines on account, $54.85; August 31, pay store rent, $30, and clerk hire, $35.

2. Rule a personal account similar to the illustration of Harold Booth's account, page 200, and enter the following transactions:

September 3, you purchase from R. G. Henderson, on account, 2/10; N/60, an invoice of goods amounting to $237.40; September 9, purchase an invoice of goods from Henderson, $58.35, terms 1/15; N/2 months; September 11, purchase an invoice of merchandise from Henderson, $123.60, terms 2/5; N/30; September 12, pay the invoice of the 3d, less the discount; September 14, purchase goods amounting to $25; September 16, pay the invoice of the 11th, less the discount. What is the balance of Mr. Henderson's account? Is this an account receivable or an account payable? Is the balance a resource or a liability?

3. Rule an account similar to the illustration of B. A. Newcomber's account, page 200, and enter the following transactions:

July 2, sold R. S. Clark, terms 1/5; N/30, merchandise amounting to $57.82; July 5, sold Clark goods to the value of $23.95, terms 2/10; N/60; July 7, received payment for the invoice of the 2d, less the discount; July 9, sold Clark an invoice of $75, less a trade discount of 10%, terms 1/10; N/30; July 11, sold him a bill of goods amounting to $40; July 15, received cash for the invoice of the 5th, less the discount.

Is this account an account receivable or an account payable?

Is the balance a resource or a liability?

CHAPTER XIX

TAKING INVENTORY

At certain regular intervals an inventory is taken to determine the value of the stock on hand. Two clerks usually work together in taking the inventory. One counts the number of items of each kind, and reads aloud the cost price marked on the goods. The second clerk records these facts. The inventory is then sent to the office, where the value of each item is extended and the total value of the stock on hand is determined.

Inventories are entered in various forms, depending upon the details of information desired.

173. Periodic Inventories. A simple form like the following may be used:

QUANTITY	NAME OF ITEM	COST		COST EXTENSION	
19 bbl.	Herald Flour	5	60	106	40
345 #	Java Coffee		26	89	70
28½ #	Ceylon Tea		43	12	26

Written Work

Rule an inventory similar to the preceding form, enter the following items, find the cost of each item, and the total cost.

This is one of several inventory sheets used in taking stock in a grocery.

87	lb.	Mexican Java Coffee	lb.	$ .23½
136	lb.	Ceylon Tea	lb.	.42
167	pkg.	Half-pounds Ceylon Tea	lb.	.47
2	cases	Boneless Herring (4 doz.)	doz.	1.10

83	lb.	Salt Mackerel	lb.	$ .22
$6\frac{5}{12}$	doz.	Cans Corn	doz.	.85
$4\frac{11}{12}$	doz.	Cans Tomatoes	doz.	.95
$8\frac{3}{4}$	doz.	Cans Peas	doz.	1.25
$9\frac{1}{6}$	doz.	Cans Beans	doz.	1.35
$3\frac{5}{6}$	doz.	Bottles Olives	doz.	1.00
$4\frac{1}{3}$	doz.	Bottles Olives (18 oz.)	doz.	3.20
$\frac{7}{12}$	doz.	Bottles Stuffed Olives (10 oz.)	doz.	$.82\frac{1}{2}$
345	lb.	Evaporated Apricots	lb.	$.08\frac{3}{4}$
163	lb.	Evaporated Apples	lb.	$.07\frac{1}{4}$
109	lb.	Dried Prunes	lb.	$.05\frac{1}{2}$
230	lb.	Dried Peaches	lb.	$.07\frac{3}{4}$
67	lb.	Seedless Raisins	lb.	$.09\frac{1}{2}$
1	case	Currants (36 lb.)	lb.	$.08\frac{1}{2}$

A more elaborate form of inventory similar to the following may be used when desired.

Add	Deduct	Lot No.	Size	Name and Quantity	Cost Price	Cost Extension	% Dep.	Loss Assumed
		A 42	6	3 Pairs Mens Gun Metal Oxfords	2 80	8 40		
	1	A 42	6½	4 Pairs do	2 80	8 40		
4		A 42	7	5 Pairs do	2 80	25 20		
		A 42	7½	6 Pairs do	2 80	16 80		
		A 42	8	2 Pairs do	2 80	5 60		
		T 0	[illegible]	3 Pairs Mens Slucher Oxfords	3 10	9 30	10%	93

174. Explanation of Form. The Lot Number is a number given an article by either the manufacturer or the merchant. Since different styles of goods are given different lot numbers, this number may be entered in the inventory in place of a description.

The "size" column is used to show the different sizes of an article in stock. As illustrated in the model inventory, each size of an article may appear on a separate line, or all sizes may be entered on the same line. When the latter method is followed, the quantity of a certain size is indicated by writing the size below a short line, and the number of articles of that size above the line. Thus, $\frac{3}{7}$, $\frac{5}{8}$, means 3 No. 7's, 5 No. 8's.

When an error is made in the count, this error may be corrected by using the "Add" or "Deduct" columns. These columns are also used when goods are purchased or sold while the inventory is being taken. By adding goods purchased, and deducting goods sold during the taking of inventory, the value of stock at the end of the inventory may be determined.

To find the cost extension, multiply the cost by the number of items plus the number of items in the "Add" column or minus those in the "Deduct" column.

If goods become shopworn or out of style, if the market price has decreased, or if, for any other reason, the value has become less than the cost marked on the goods and entered in the inventory, it is necessary to make some allowance for this depreciation in value.

Perhaps an article is worth only half its cost price. In that case a 50% depreciation should be entered in the "% Depreciation" column. Then 50% of the cost extension for that item is entered in the "Loss Assumed" column, and the total of this column is subtracted from the total of the "Cost Extension" column to determine the actual value of the stock on hand.

Oral Work

1. What does 25% depreciation mean? 10% depreciation? 20% depreciation?

2. If stock is damaged so that it is worth only $\frac{3}{4}$ of its original cost, what per cent depreciation should be entered?

3. Goods are worth $\frac{2}{3}$ of their original cost. What is the per cent depreciation?

4. Goods are worth $\frac{4}{5}$ of their original cost. What is the per cent depreciation?

5. Goods are worth $\frac{7}{8}$ of their original cost. What is the per cent depreciation?

6. An article purchased for \$3.20 can now be bought for \$2.80. What per cent depreciation should be entered in the "per cent depreciation" column?

7. What would be the amount of the loss assumed on 27 of the articles mentioned in problem 6?

Written Work

Rule an inventory form similar to the one on page 204 and enter the following items which form a portion of the inventory of a shoe stock. Enter one size to a line, find the value of each item listed, and the total value of the stock shown on the page after making corrections for stock returned, for sales, and for depreciation.

Lot No. 219 B; $\frac{2}{6}$, $\frac{1}{6\frac{1}{2}}$, $\frac{3}{7}$, $\frac{4}{7\frac{1}{2}}$, $\frac{2}{8}$, $\frac{2}{8\frac{1}{2}}$. Men's Tan Oxfords, \$3.15 per pair.

Lot No. 712; $\frac{2}{6}$, $\frac{3}{6\frac{1}{2}}$, $\frac{4}{7}$, $\frac{4}{7\frac{1}{2}}$, $\frac{3}{8}$, $\frac{4}{8\frac{1}{2}}$, $\frac{2}{9}$. Men's Vici Kid Bluchers, \$2.90 per pair.

Lot No. 322 A; $\frac{1}{6}$, $\frac{1}{6\frac{1}{2}}$, $\frac{2}{7}$. Romeos at \$1.90 per pair.

Lot No. 618 D1; $\frac{2}{7}$, $\frac{3}{7\frac{1}{2}}$, $\frac{3}{8}$, $\frac{2}{8\frac{1}{2}}$, $\frac{2}{9}$, $\frac{1}{9\frac{1}{2}}$. Men's Russian Calf Boots, \$3.20 per pair.

One pair of Lot No. 712, Size 7, taken home by a customer before taking inventory, was returned after Lot No. 712 was listed.

During stock taking, the following sales were made:

1 pair Lot No. 712, size $7\frac{1}{2}$.
1 pair Lot No. 618 D1, size $7\frac{1}{2}$.
1 pair Lot No. 618 D1, size $9\frac{1}{2}$.

Lot No. 322 A can now be purchased for \$1.75.

175. Perpetual Inventory. In some lines of business a perpetual inventory is kept. It is also called a Stock Record. When such

a record is kept, a book is required for the purpose, one page being devoted to each item carried in stock. Both purchases and sales are recorded at cost price. The following model illustrates a form of perpetual inventory.

No. A 603 Brace Castings

Danger Point 15

Date		Purchases			Sales		Balance	
1915		No.	Cost	Value	No.	Value	No.	Value
July	8	60	18	10 80			60	10 80
"	17				14	2 52	46	8 28
"	22				26	4 68	20	3 60
"	30	80	18	14 40				18 00
Aug.	3				25	4 50		13 50

176. Value of Perpetual Inventory. Considerable labor is required to maintain a perpetual inventory, but it is valuable for several reasons.

When stock is kept in warehouses or in storerooms at some distance from the salesrooms, the perpetual inventory is a great convenience since it shows without delay the amount of stock on hand.

A "danger point" is fixed for each item. When the balance on hand has decreased to this danger point, a new supply is purchased.

An inventory may be taken at periodical intervals, and the quantities shown compared with the stock record. If the inventory and the stock record do not agree, an investigation may be made to discover the cause of the discrepancy.

Written Work

Rule a page of a stock record similar to the model on page 207, and enter the following facts, making a perpetual inventory of the stock of Solvay Coke.

PURCHASES

October 22, 54,200#	at $4.45 per ton
November 18, 57,600#	at $4.45 per ton
December 7, 52,800#	at $4.45 per ton

SALES

October 29, 2 tons; October 30, 5 tons; October 31, 7500#; November 2, 9500#; November 5, 7 tons; November 17, 2 tons; November 19, 12 tons; November 20, 7000#; November 23, 4 tons; November 24, 9000#; November 30, 2 tons; December 9, 13,000#; December 11, 5 tons.

CHAPTER XX

GROSS TRADING PROFIT

177. Definitions. **Gross Trading Profit** is the difference between the selling price and the cost price.

Net Profit is found by subtracting the expenses and losses of the business from the gross profit.

178. The Per Cent of Gross Trading Profit is found by dividing the gross profit by the net sales.

If goods are sold at less than their cost, a loss results. The per cent of loss is found by dividing the loss by the net sales.

Oral Work

1. A hat which cost \$2 is sold for \$3. What is the profit? What is the per cent of profit?

2. A set of books cost a dealer \$20. In order to sell them the dealer lost \$2.00. What was the per cent of loss?

3. Through an error in an advertisement a merchant was obliged to sell some parasols at $16\frac{2}{3}\%$ less than their cost. He lost 30 cents on each parasol. What did they cost, and what was the selling price? At what price should they have been sold to gain 10% of the cost?

Written Work

1. Mr. Fisher bought a carriage for \$55 and sold it for \$75. What was the gross profit? What was the per cent of gross profit?

2. During a certain year a merchant purchased goods which cost him \$14,000. At the close of the year this stock inventoried \$3000. What was the cost of the goods sold?

His sales for the year were \$14,080. What was his gross profit? What was the per cent of gross profit?

3. The inventory of a dry goods merchant's stock at the beginning of the year was $3560. The purchases during the year were $28,265. At the close of the year the inventory showed a stock of $3940 unsold. What was the cost of the goods sold?

His sales for the year were $31,673. What was the gross profit? What was the per cent of gross profit?

179. Buying Expenses. In computing the cost of goods purchased, it is customary to add to the wholesale price, the freight, drayage, and all other expenses incurred in getting the goods on the shelves ready to sell. These expenses are called *buying expenses.*

Written Work

1. Mr. Tracey bought the lumber business formerly conducted by Mr. Boyce. The stock on hand inventoried $5280. Mr. Tracey's purchases during the year were $16,385. The freight was $236, the cost of labor in placing lumber in the yards and all other buying expenses $385. At the end of the year the stock inventoried $6125. What was the cost of the goods sold during the year?

If the sales during the year were $19,245, what was the gross profit and per cent of gross profit?

2. Complete the following table:

Inventory, Jan. 1, 19—	Purchases	Buying Expenses	Total Cost of Purchases	Inventory, Dec. 31, 19—	Cost of Goods Sold	Sales	Profit or Loss	Per Cent Profit or Loss
3125 75	16,294 90	370 25		4685 60		18,495 40		
5428 80	23,926 56	627 90		5626 35		33,756 20		
1290 00	6,754 00	271 25		2140 00		8,325 75		
4728 50	12,398 80	1417 25		1265 70		13,215 75		
3680 00	15,927 00	485 00		2726 00		14,850 00		

Enter profit and per cent of profit in black ink.
Enter loss and per cent of loss in red ink.

CHAPTER XXI

INTEREST

If you should rent a house, you would agree to pay a certain sum for the use of the house and you would probably sign an agreement to pay this rent. If you should borrow a sum of money, you would probably agree to pay the lender a certain sum for the use of the money and you would probably be required to sign an agreement to repay with interest the sum borrowed.

180. Terms Used. Money paid for the use of money is called **interest.** Interest is usually computed as a certain per cent of the amount borrowed. The per cent of interest is called the **rate** and the amount borrowed the **principal.** The statement that a man borrowed \$3000 at 6 %, means that he must pay 6 % of \$3000, or \$180, each year for the use of the \$3000. At the end of a year he would owe the lender \$3000 plus the interest (\$180), or \$3180. The sum of the principal and the interest is called the **amount.**

181. Promissory Notes.

It is the custom in business for the man who borrows money to give a written promise to repay the sum borrowed. Such a promise is called a **promissory note.**

\$ 50 00 — New York, Oct 17th 1915

Two months — after date I promise to pay to

the order of Henry G Heath

Fifty and 00/100 — Dollars

at First National Bank, with interest @ 6%

Value received

No. 3 — Due Dec 17th/15 — C M Freeman

Promissory Note

If the borrower is a person of small means, or if, for any other reason, it may be unsafe to loan him money, he may be requested to have some person of recognized financial standing sign the note with him. Such notes are either **joint notes**, or **joint and several notes**, depending upon the wording.

$400.00 New York, July 15th 1915

Thirty days after date we jointly and severally promise to pay to the order of Martin Haskin

Four Hundred ——— Dollars

at his office

Value received Fred G. Hamilton

No. Due Stephen Merrill

JOINT AND SEVERAL NOTE

$1000.00 Aurora, Ill., April 13, 1915

Ninety days after date I promise to pay to the order of First National Bank of Aurora, Ill.

One thousand ——— Dollars

Without defalcation, value received with interest at 6%

And further, do hereby empower any Attorney of any Court of Record within the United States or elsewhere to appear for me and after one or more declarations filed confess judgment against me in of any term for the above sum with costs of suit and Attorney's commission of ten per cent for collection and release of all errors and without stay of execution and inquisition and extension upon any levy on real estate is hereby waived and said mention agreed to and the exemption of personal property from levy and sale on any execution hereon, is also hereby expressly waived, and no benefit of exemption be claimed under said by virtue of any exemption law now in force or which may be hereafter passed.

Witness my hand and seal A. B. Jones (SEAL)

Due July 12, 1915 (SEAL)

JUDGMENT NOTE

182. To find interest at 6 % for 6, 60, 600, or 6000 days.

The interest on $1 for 1 year at 6 % is $.06

The interest on $1 for 2 months (60 da.) is .01

The interest on $1 for 6 days ($\frac{1}{10}$ of 60 da.) is .001

Thus, to find the interest on $1 for 60 days at 6 %, move the

decimal point two places to the left; to find the interest for 6 days, move the point three places to the left. Since this is true if the principal is $1, it will be true of any principal, and we have the following rule:

Given any principal, to find the interest at 6 %.

For 6 days, move the decimal point three places to the left; for 60 days, two places to the left; and for 600 days, one place to the left. For 6000 days the interest will be the same as the principal.

Oral Work

State the interest on each of the following principals at 6 %, for 6, 60, 600, and 6000 days:

1. $1000. 2. $350. 3. $125.60. 4. $3620.
5. $12.20. 6. $325.50. 7. $6.

Find the interest at 6 % on:

8. $245 for 6 days. 9. $27 for 6 days.
10. $37.50 for 60 days. 11. $12.50 for 6 days.

183. To find interest at 6 % for fractions or multiples of 6, 60, 600, or 6000 days.

The time is not always 6, 60, or 600 days, yet the above method can be easily used when the time is an easy fraction or an exact multiple of 6, 60, 600, or 6000 days.

Examples. 1. Find the interest on $500 for 12 days at 6 %.

SOLUTION. $.50 = the interest for 6 days.
$2 \times \$.50 = \$1$, the interest for 12 days.

2. Find the interest on $700 for 40 days at 6 %.

SOLUTION. $7 = the interest for 60 days.
$\frac{2}{3}$ of $7 = $4.67, the interest for 40 days.

3. Find the interest on $300 for 3 months at 6 %.

SOLUTION. $3 = the interest for 2 months (60 days).
$3 \div 2 = \$1.50$, the interest for 1 month.
$1.50 \times 3 = \$4.50$, the interest for 3 months.

Written Work

Rule a form similar to the following model; compute the interest at 6 % on the various principals for the different times.

Principals	24 Da.	30 Da. (1 Mo.)	40 Da.	80 Da.	90 Da. (3 Mo.)	18 Da.	50 Da.	70 Da.	4 Mo.	6 Mo.
$125.60										
28.95										
378.45										
500.00										
960.00										
96.00										
9.60										
475.00										
4.75										
30.00										
146.00										
419.50										
395.00										
25,000.00										

When the time is not an easy fraction or an exact multiple of 6, 60, or 600 days, an adaptation of the method shown in the last exercise may be used.

Examples. 1. Find the interest on $500 for 99 days at 6 %.

Solution. $5.00 = the interest for 60 days.
2.50 = the interest for 30 days.
.50 = the interest for 6 days.
.25 = the interest for 3 days.
$8.25 = the interest for 99 days.

2. Find the interest on $1000 for 82 days at 6 %.

Solution. $10.00 = the interest for 60 days.
$3.33\frac{1}{3}$ = the interest for 20 days.
$.33\frac{1}{3}$ = the interest for 2 days.
$13.67 = the interest for 82 days.

Oral Work

How would you find the interest on any principal for the following number of days at 6 %:

3, 4, 5, 7, 8, 9, 36, 66, 69?

Written Work

Enter the interest at 6 % in the proper place on a form ruled like the following model :

	$845.00	$632.00	$1296.00	$5483.00	$74.60	$143.40	$746.95	$25.00	$130.00
10 da.									
66 da.									
63 da.									
68 da.									
75 da.									
80 da.									
85 da.									
96 da.									
98 da.									
45 da.									
82 da.									
28 da.									

184. To find interest at 6 % for any number of days.

Sometimes the method of adding the interest for different fractions or multiples of 6 and 60 days cannot conveniently be used. In such cases

Point off three places in the principal; this will give the interest for six days. Multiply by the number of days; this will give the interest for six times the number of days. Divide by six; this will give the interest for the required number of days.

Example. Find the interest on $500 for 19 days at 6 %.

SOLUTION. $.50 = the interest for 6 days.
$.50 × 19 = $9.50, the interest for 19 × 6 days.
$9.50 ÷ 6 = $1.58, the interest for 19 days.

Written Work

Enter the interest at 6 % on the various principals for the different times in the proper place on a form ruled like the following model:

	$435.00	$127.90	$213.65	$472.90	$123.50	$12.35
23 da.						
29 da.						
37 da.						
19 da.						
25 da.						
73 da.						
47 da.						
81 da.						
114 da.						
95 da.						
61 da.						
77 da.						
183 da.						
211 da.						

185. Interchanging Principal and Time. There is a short method which may sometimes be used when the days are not exact multiples of 6, 60, 600, or 6000.

The interest is the same in each of the following cases:

$600.00 for 29 days at 6%, and
$ 29.00 for 600 days at 6%.

It is much easier to compute the interest in the latter case, because all that is necessary is to point off one place in the principal, giving the amount of interest immediately, $2.90.

Therefore, when the days are not exact multiples or easy fractions of 6, 60, 600, or 6000, and when the principal *is* an exact multiple or easy fraction of one of these numbers, interchange the number of dollars and days, and proceed as before.

Examples. 1. Find the interest on $360.00 for 83 days at 6%.

SOLUTION. Interchanging, we have $83.00 for 360 days at 6%.
Pointing off two places, $.83 interest for 60 days.
Multiply by 6 for 360 days, and the result is $4.98.

2. Find the interest on $200.00 for 47 days at 6%.

SOLUTION. Interchanging, we have $47.00 for 200 days at 6%.
Point off one place for 600 days, $4.70.
Divide by 3 for 200 days, $1.57.

Oral Work

How would you find the interest on the following at 6 %?

1. $700.00 for 71 days.
2. $150.00 for 59 days.
3. $420.00 for 113 days.
4. $30.00 for 57 days.
5. $66.00 for 171 days.

Written Work

Compute and enter the interest at 6 % on a form ruled like the following model:

	117 Da.	84 Da.	161 Da.	59 Da.	74 Da.	3 Mo. 7 Da.
$ 300 00						
1200 00						
240 00						
4200 00						
150 00						
72 00						
450 00						
54 00						
6000 00						
5000 00						

186. To find the interest when the rate is other than 6 %.

First find the interest at 6 %, then take $\frac{1}{6}$ of the result; this will give the interest at 1 %. Multiply this quotient by the given rate.

The following short methods are of value in computing interest:
If the rate is 7 %, add $\frac{1}{6}$ of the interest at 6 %.
If the rate is 5 %, subtract $\frac{1}{6}$ of the interest at 6 %.
If the rate is $7\frac{1}{2}$ %, add $\frac{1}{4}$ of the interest at 6 %.
If the rate is $4\frac{1}{2}$ %, subtract $\frac{1}{4}$ of the interest at 6 %.

Oral Work

How would you find the interest at 4 %? $5\frac{1}{2}$ %? $6\frac{1}{2}$ %? 8 %? 9 %?

Written Work

Complete the following blank:

SIMPLE INTEREST

RATE	36 DA.	45 DA.	63 DA.	19 DA.	47 DA.	184 DA.	2 MO. 7 DA.
	$436.50	$75.80	$174.30	$66.00	$1748.60	$73.90	$400.00
6 %							
8 %							
5 %							
$4\frac{1}{2}$ %							
$3\frac{1}{2}$ %							

187. To find the interest when the time is stated in years, months, and days.

In the preceding problems the time has been stated in days and months. When the time has been stated in months, we have reduced it to days, but when the time is stated in years, months, and days, it is too laborious to reduce it to days, and the following method should be used.

Find the interest for the number of years at the given per cent. To this result add the interest for the given number of months and days.

Example. Find the interest on $375 for 2 yr. 3 mo. 21 da. at 5 %.

SOLUTION. The interest for 1 yr. is 5 % of the principal.
The interest for 2 yr. is 10 % of the principal.
10 % of $375 = $37.50.
The interest on $375 for 3 mo. 21 da. is $5.78.
Therefore, the interest for 2 yr. 3 mo. 21 da. is $43.28.

Written Work

Rule a blank like the following model. Compute and enter the interest.

Complete the blank:

SIMPLE INTEREST

TIME			$124.00 6%		$87.62 5%		$1245.75 3½%		$463.80 4%		$2000.00 5½%	
Years	Months	Days										
1	5	24										
1	8	12										
2	4	18										
3	8	6										
4	10	12										
10	2	7										
8	6	2										
5	7	9										
3	4	15										
2	7	28										
3	3	20										
4	7	27										
9	1	19										

188. To compute the time between two dates. In all the problems given thus far, the time has been stated. You will often be called upon to compute the time. In such cases you will be given the date when the note was made and the date when it is due (date of maturity).

There are two methods of computing the time between dates:

First Method. Counting the actual number of days. (Not used if the interval is more than one year.)

Second Method. Compound subtraction. (May be used for any interval.)

a. **To find the time by counting the actual number of days.**

Example. Find the number of days between February 15, 1915, and May 24, 1915.

SOLUTION.

February has 28 days, of which 15 have expired, leaving in February	13 days
March has	31 days
April has	30 days
The note matures on May	24
Total	98 days

From this problem we derive the following rule:

Add the number of days remaining in the month after the day the note was made, the exact number of days in each complete month, and the number of days through which the note is to run in the last month.

Written Work

Find the exact number of days between the dates in the table:

Made	Due					
	Oct. 26	Nov. 13	Dec. 27	Jan. 3	Jan. 17	Feb. 26
Oct. 13						
Sept. 27						
Aug. 18						
July 16						
July 3						
May 17						
May 1						

b. **To find the interest period by compound subtraction of dates.**

Example. Find the time between May 18, 1910, and February 16, 1913

Solution.

	1913	2	16
	1910	5	18

Since 18 cannot be subtracted from 16, and since 5 cannot be subtracted from 2, reduce 1 month to days, and 1 year to months.

	1912	13	46
	1910	5	18
	2 yr.	8 mo.	28 da.

Written Work

Find the time by compound subtraction. The figures in the left-hand column are the dates on which the notes were given. The dates at the top are the dates of maturity.

From	To			
	Maturity 10–22–1913	Maturity 6–15–1914	Maturity 11–14–1914	Maturity 5–20–1915
6–17–1913				
8– 5–1913				
9–25–1912				
11– 6–1912				
1–19–1910				
2–26–1909				
1–30–1912				
7–14–1913				
10–11–1906				
12–31–1912				

The following table may be used in finding the exact number of days between any two dates :

For illustration, the time from Apr. 7 to Nov. 7 is found to be 214 da. From Sept. 3 to Jan. 3 is found to be 122 da. To find the time from Sept. 3 to Jan. 18, add 15 da. to 122 da.

From Any Day of	To the Same Day of the Next											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January . .	365	31	59	90	120	151	181	212	243	273	304	334
February . .	334	365	28	59	89	120	150	181	212	242	273	303
March . . .	306	337	365	31	61	92	122	153	184	214	245	275
April . . .	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June . . .	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August . . .	153	184	212	243	273	304	334	365	31	61	92	122
September . .	122	153	181	212	242	273	303	334	365	30	61	91
October . . .	92	123	151	182	212	243	273	304	335	365	31	61
November . .	61	92	120	151	181	212	242	273	304	334	365	30
December . .	31	62	90	121	151	182	212	243	274	304	335	365

Review

1. In the following exercise, enter the time (in red ink[1]) on the upper of the two lines, and the interest (in black ink) on the lower line.

[1] At teacher's discretion.

If the time is less than a year, compute the actual number of days.

If the time is more than a year, find the interval by compound subtraction, and state it in years, months, and days.

From	Rate 6%			
	$216.80	$625.00	$923.40	$87.86
	To			
	9-29-1914	11-26-1914	12-24-1914	1-26-1915
9-22-1914				
6-24-1914				
4- 2-1914				
2-11-1914				
11-13-1913				
8- 6-1913				
5-22-1913				
4-25-1912				
7-13-1910				

2. Complete the following, entering the time on the upper of the two lines, and the interest on the lower.

From	5%	4½%	7%	7½%
	To			
	May 26, 1914	Aug. 17, 1914	Oct. 9, 1914	Feb. 10, 1915
Principal, $219.80 April 22, 1914				
Principal, $326.00 May 17, 1914				
Principal, $80.00 September 19, 1913				
Principal, $260.00 May 14, 1913				

Written Review

Use the simplest method to find the interest on the following:

1. Interest at 6 % on $321.65 for 24 da.
2. Interest at 6 % on $523.19 for 20 da.
3. Interest at 6 % on $413.00 for 75 da.
4. Interest at 6 % on $300.00 for 57 da.
5. Interest at 5 % on $450.00 for 35 da.
6. Interest at 7½ % on $216.80 for 42 da.
7. Interest at 4 % on $625.00 for 117 da.
8. Interest at 6 % on $80.00 for 3 yr. 8 mo. 15 da.
9. Interest at 6 % on $50.00 from March 24, 1914, to May 26, 1914.
10. Interest at 7 % on $125.00 from July 19, 1911, to January 14, 1915.

189. Compound Interest. Compound interest is interest computed, at regular intervals, on the sum of the principal and any unpaid interest. In other words, as soon as interest becomes due and is unpaid, it begins to draw interest at the same rate as the principal. Compound interest is generally paid on the deposits in savings banks and is used in calculating sinking funds.

Interest may be compounded quarterly, semi-annually, annually, or at the end of any other period agreed upon. In some states the collection of compound interest is not permitted.

Example. Find the amount and the compound interest of $1200 at 6 % for two years, interest compounded semi-annually.

SOLUTION.

$1200.00	First principal
36.	Interest for 6 months
1236.	Principal at beginning of second 6-months period
37.08	Interest for second 6 months
1273.08	Principal at beginning of third period
38.19	Interest for third period
1311.27	Principal at beginning of fourth period
39.34	Interest for fourth period
$1350.61	Amount at end of two years
$1350.61	Amount at end of two years
1200.00	Principal
$150.61	Compound interest

Written Work

1. Find the compound interest on $500.00 for 5 years at 6%, interest compounded annually.

2. What is the amount and the compound interest on $7500.00 loaned for 3 years at 6%, interest compounded semi-annually?

3. Find the interest on $1200.00 loaned for 2 years at 5% compound interest, interest compounded quarterly.

Compound interest is computed with much less work by the use of tables showing the accumulation of interest and principal when $1.00 is loaned at compound interest.

190. Compound Interest Table.

Showing the amount of $1.00 compounded annually at various rates

Years	1%	1½%	2%	2½%	3%	3½%	4%	4½%	5%	6%	Years
1	1.010000	1.005000	1.020000	1.025000	1.030000	1.035000	1.040000	1.045000	1.050000	1.060000	1
2	1.020100	1.030225	1.040400	1.050625	1.060900	1.071225	1.081600	1.092025	1.102500	1.123600	2
3	1.030301	1.045678	1.061208	1.076891	1.092727	1.108718	1.124864	1.141166	1.157625	1.191016	3
4	1.040604	1.061364	1.082432	1.103818	1.125509	1.147523	1.169859	1.192519	1.215506	1.262477	4
5	1.051010	1.077284	1.104081	1.131408	1.159274	1.187686	1.216653	1.246182	1.276282	1.338226	5
6	1.061520	1.093443	1.126162	1.159693	1.194052	1.229255	1.265319	1.302260	1.340096	1.418519	6
7	1.072135	1.109845	1.148686	1.188686	1.229874	1.272279	1.315932	1.360862	1.407100	1.503630	7
8	1.082857	1.126493	1.171659	1.218403	1.266770	1.316809	1.368569	1.422101	1.477455	1.593848	8
9	1.093685	1.143390	1.195093	1.248863	1.304773	1.362897	1.423312	1.486095	1.551328	1.689479	9
10	1.104622	1.160541	1.218994	1.280085	1.343916	1.410599	1.480244	1.552969	1.628895	1.790848	10
11	1.115668	1.177949	1.243374	1.312087	1.384234	1.459970	1.539454	1.622853	1.710339	1.898299	11
12	1.126825	1.195618	1.268242	1.344889	1.425761	1.511069	1.601032	1.695881	1.795856	2.012197	12
13	1.138093	1.213552	1.293607	1.378511	1.468534	1.563956	1.665074	1.772196	1.885649	2.132928	13
14	1.149474	1.231756	1.319479	1.412974	1.512590	1.618695	1.731676	1.851945	1.979932	2.260904	14
15	1.160969	1.250232	1.345868	1.448298	1.557967	1.675349	1.800944	1.935282	2.078928	2.396558	15
16	1.172579	1.268986	1.372786	1.484506	1.604706	1.733986	1.872981	2.022370	2.182875	2.540352	16
17	1.184304	1.288020	1.400241	1.521618	1.652848	1.794676	1.947901	2.113377	2.292018	2.692773	17
18	1.196148	1.307341	1.428246	1.559659	1.702433	1.857489	2.025817	2.208479	2.406619	2.854339	18
19	1.208109	1.326951	1.456811	1.598650	1.753506	1.922501	2.106849	2.307860	2.526950	3.025600	19
20	1.220190	1.346855	1.485947	1.638616	1.806111	1.989789	2.191123	2.411714	2.653298	3.207136	20
21	1.232392	1.367058	1.515666	1.679582	1.860295	2.059431	2.278768	2.520241	2.785963	3.399564	21
22	1.244716	1.387564	1.545980	1.721571	1.916103	2.131512	2.369919	2.633652	2.925261	3.603537	22
23	1.257163	1.408377	1.576899	1.764611	1.973587	2.206114	2.464716	2.752166	3.071524	3.819750	23
24	1.269735	1.429503	1.608437	1.808726	2.032794	2.283328	2.563304	2.876014	3.225100	4.048935	24
25	1.282432	1.450945	1.640606	1.853944	2.093778	2.363245	2.665836	3.005434	3.386355	4.291871	25

Example. Find the compound interest on $3562.80 for 4 years at 6%, interest compounded annually.

SOLUTION. $1.00 compounded annually at 6% for 4 years amounts to $1.262477, as shown by the table.

$3562.80 \times \$1.262477 = \$4497.95$, amount of $3562.80 compounded annually for 4 years at 6%.

$\$4497.95 - \$3562.80 = \$935.15$, compound interest.

When interest is compounded semi-annually, take $\frac{1}{2}$ the rate for twice the time.

When interest is compounded quarterly, take $\frac{1}{4}$ the rate for 4 times the time.

Example. What is the compound interest on $5000.00 at 8% for 3 years 6 months, interest compounded quarterly?

SOLUTION. $\frac{1}{4}$ of 8% = 2%.

4 times 3 years 6 mo. = 14 years.

The amount of $1.00 compounded at 2% for 14 years is $1.319479.

$5000 \times \$1.319479 = \$6597.40$.

$\$6597.40 - \$5000.00 = \$1597.40$, compound interest.

Written Work

1. Find the compound interest on $2500.00 for 5 years at 5%, interest compounded annually.

2. $5000.00 was invested at 4%, interest compounded semi-annually. To what sum did this investment amount in 7 years?

3. What was the compound interest on a loan of $3750.00, made June 1, 1909, and due December 1, 1914, at 6%, interest compounded quarterly?

4. What sum must be invested at 5% compound interest to amount, in 7 years, to $3249.84, if the interest is compounded semi-annually?

CHAPTER XXII

SAVINGS BANKS

191. Checking and Savings Accounts. Most banks receive deposits under two classes of accounts, checking accounts and savings accounts.

Money deposited in checking accounts can be drawn out by checks payable to the depositor or to any other party. These checks are payable on presentation at the bank. The cash on deposit usually does not bear interest.

Money deposited in a savings account cannot be drawn out by a check payable to any person other than the depositor himself. The law usually provides that the bank may require notice of from 10 to 60 days before paying money from a savings account. Banks rarely take advantage of this privilege, however. The cash on deposit bears interest at some rate fixed by the bank. 3%, $3\frac{1}{2}$%, and 4% are common rates.

192. Computing Interest on Savings Accounts. Interest is computed on the **smallest balance** which the depositor leaves in the bank during the entire time between fixed days. These fixed days are called **interest days**, and the time between the interest days is called the **interest term**.

Some banks pay interest on dollars only, and ignore the cents. Interest which is not withdrawn by the depositor is added to his account, and draws interest the same as a deposit. Thus, savings banks really pay compound interest.

Example. The interest days of the Snowden Savings Bank are January 1, April 1, July 1, and October 1. On each of these interest days, interest at the rate of 4% per annum is computed on the smallest quarterly balance. The account of W. M. Scott is shown on the following page.

W. M. Scott

Date		Deposits	Interest	Withdrawals	Balance
1915					
Feb.	7	120 00			120 00
April	20	300 00			420 00
April	30			100 00	320 00
July	1		1 20		321 20
July	15	50 00			371 20
Sept.	17	75 00			446 20
Oct.	1		3 21		449 41
Nov.	16			80 00	369 41
1916					
Jan.	1		3 69		373 10

Explanation. The first interest term was from Jan. 1 to April 1, but since Mr. Scott made no deposit until Feb. 7, the smallest balance on deposit during the entire interest term was $0.00, and, therefore, no interest was added on April 1.

The second interest term was from April 1 to July 1. The smallest balance on deposit during the entire term was $120.00 (the balance on April 1). Interest, $1.20.

The third interest term was from July 1 to Oct. 1. Smallest balance $321.20. Interest, $3.21.

The fourth interest term was from Oct. 1 to Jan. 1. Smallest balance $369.41. Interest, $3.69.

193. Interest Terms and Dates of Adding Interest. Interest terms are not of uniform length among the various banks of the country. Some banks compute interest on monthly balances; some on quarterly balances, and some on semi-annual balances.

Some banks add interest quarterly and some add it semi-annually. The rules applying to the interest computations of any bank may usually be found in the by-laws of the bank, printed in the depositor's pass book.

Examples. 1. Suppose the Snowden Savings Bank computed interest on quarterly balances, but added interest semi-annually. W. M. Scott's account would appear as follows:

W. M. Scott

Date		Deposits	Interest	Withdrawals	Balance
1915					
Feb.	7	120 00			20 00
April	20	300 00			420 00
April	30			100 00	320 00
July	1		1 20		321 20
July	15	50 00			371 20
Sept.	17	75 00			446 20
Nov.	16			80 00	366 20
1916					
Jan.	1		6 87		373 07

		Smallest Balance	Quarterly Interest	Semi-annual Dividend of Int.
Explanation.	First Quarter	$ 0.00	$ 0.00	
	Second Quarter	120.00	1.20	$ 1.20
	Third Quarter	321.20	3.21	
	Fourth Quarter	366.20	3.66	6.87

2. Suppose the Snowden Savings Bank computed interest on the monthly balance and added interest quarterly. Scott's account would appear as follows:

Explanation.

Month	Monthly Balance	1 Month Interest ⅓ of 1%	Quarterly Dividend of Interest
Feb.	$000 00	$ 0 00	
March	120 00	40	$ 0 40
April	120 40	40	
May	320 40	1 06	
June	320 40	1 06	2 52
July	322 92	1 07	
Aug.	372 92	1 24	
Sept.	372 92	1 24	3 55
Oct.	451 47	1 50	
Nov.	371 47	1 23	
Dec.	371 47	1 23	3 96

In computing interest on savings accounts, cents in the principal are dropped. Fractions of cents in interest are dropped.

Written Work

1. Check the computations in each of the illustrations above.

2. Prepare W. M. Scott's account on the supposition that the bank computes interest on monthly balances and adds interest semi-annually, Jan. 1 and July 1.

3. Prepare W. M. Scott's account on the supposition that the bank computes interest on semi-annual balances and adds interest semi-annually. Remember that interest is paid on the smallest balance on deposit during the *entire* interest term.

4. Which of the methods employed in this section is most profitable for the depositor?

5. Suppose you deposited $30 in a savings bank and left it to draw compound interest for 5 years at 4%, interest compounded semi-annually. How much could you withdraw at the end of the fifth year?

6. Suppose you had loaned the $30 for 5 years at 4% simple interest. What would the interest have been for five years?

7. Compare the simple and compound interest in Problems 5 and 6.

8. A man deposits $1200 in a savings bank to be used to send his boy to college. The savings bank pays 4% interest, compounded semi-annually. The deposit was made on the boy's tenth birthday. What amount will he have to his credit on the boy's eighteenth birthday if no withdrawals and no other deposits are made?

9. Make accounts with D. O. Dorman; one for each of the following methods of declaring interest. Rate 4%.

a. Interest computed on monthly balances. Dividends added Jan. 1, April 1, July 1, and Oct. 1.

b. Interest computed on quarterly balances. Interest days same as above.

c. Interest computed on quarterly balances. Dividends added Jan. 1 and July 1.

Date	Deposits	Withdrawals
1915		
Jan. 16	$500.00	
Feb. 19	700.00	
Mar. 1		$ 25.00
Mar. 8	60.00	
June 10	125.00	
July 12		60.00
Aug. 5	200.00	
Oct. 19	145.00	
Nov. 20		125.00

194. Postal Savings Banks. The United States Government has provided a savings bank system operated in conjunction with the postal service, whereby savings may be deposited at interest with the security of the United States Government.

Deposits. Any person ten years of age or over may become a depositor. Sums less than $1.00 cannot be deposited. No person can deposit more than $100.00 in any one calendar month, nor have a balance at any time of more than $500.00, exclusive of interest. Depositors receive a postal savings certificate for the amount of each deposit. Interest is allowed on these certificates at the rate of two per cent for each *full year* that the money remains on deposit, *beginning with the first day of the month following the one in which it was deposited.*

Withdrawals. Money may be withdrawn by surrendering to the officer where the deposit was made the savings certificates covering the desired amount.

Bonds. Under certain conditions a depositor may surrender certificates in amounts of $20.00 or any multiple of $20.00, up to and including $500.00 and receive in return government bonds bearing interest at $2\frac{1}{2}\%$ per year. These postal savings bonds may be held in addition to the $500.00 deposit allowed to one depositor.

Written Work

1. How much interest would a man receive if he deposited $15.00 in a postal savings bank on March 1, 1915, and withdrew it three months later?

2. How much interest would he receive if he withdrew the money on March 11, 1916?

3. A man made the following deposits in a postal savings bank:

May 20, 1915	$15.00
June 17, 1915	6.00
August 11, 1915	26.00
October 29, 1915	17.00

When would he be entitled to the full year's interest on each of his deposits, and how much interest would he receive all together?

CHAPTER XXIII

DISCOUNTING NOTES AND OTHER COMMERCIAL PAPER

195. Borrowing from a Bank. Banks prefer to have the borrower pay the interest in advance. The interest may be paid in cash, or the bank may subtract it from the face of the note, giving the borrower the difference, which is called the **proceeds.**

If you gave a bank the following note:

$100.00 CHICAGO, ILL., March, 2, 1915.

Ninety days after date, I promise to pay to the order of the Bowen National Bank One Hundred Dollars.

Your Signature

the bank would deduct ninety days' interest, $1.50, and you would receive the proceeds, $98.50.

NOTE. There is no promise to pay interest, because the interest is prepaid. Banks charge any rate they choose, within the limits set by the state law. In this text, when no rate is mentioned use 6 %.

The above illustration shows the method of **discounting a note.**

196. Terms Used in Bank Discount. The following terms are frequently used, and you should become familiar with them:

Maturity means the date on which the note is due.

The **value** of the note is the amount due at maturity.

If the note draws interest, the *value* is the sum of the principal and the interest. Otherwise, the *value* is the face of the note.

The **discount period** is the exact number of days between the date of discounting the paper and its maturity.

The **bank discount** is the simple interest on the value for the discount period.

The difference between the value and the bank discount is called the *proceeds.*

Oral Work

In the illustration on the previous page what is:

The value? The maturity? The discount period?
The bank discount? The proceeds?

197. To find the bank discount and the proceeds.

To compute the bank discount, compute the simple interest on the value of the note for the discount period.

To find the proceeds, subtract the bank discount from the value of the note.

Written Work

How much would you receive from the bank if you discounted the following notes?

1. A note for $500, without interest, due in 30 days, discounted at 6 %.

2. A note for $725, without interest, due in two months, discounted at 6 %.

3. A note for $85.50, without interest, due in 90 days, discounted at 7 %.

4. The following blank shows the value of several notes and the periods for which they were discounted. Compute the bank discount at 6 %, and find the proceeds.

Note Due In		Value						
		$87.00	$265.00	$962.50	$1000.00	$275.00	$765.00	$35.00
30 days	Bank discount	.44						
	Proceeds	86.56						
60 days	Bank discount							
	Proceeds							
80 days	Bank discount							
	Proceeds							
90 days	Bank discount							
	Proceeds							
120 days	Bank discount							
	Proceeds							

198. Discounting the Paper of Other Persons. If you have a note signed by some person of good financial standing, you can borrow money on it by discounting it in the same way that you would discount a note made by yourself.

Suppose John Doe had given you the following note :

$500.00 CHICAGO, ILL., January 16, 1915.

Ninety days after date I promise to pay to the order of Your Name, Five Hundred Dollars.

JOHN DOE.

By the following indorsement,

Pay to the order of
The First National Bank
(Your Name)

you transfer the note to the bank. Moreover you promise to pay the bank $500 on April 16, in case Doe fails to do so. The bank will loan you $500 minus the discount.

When you discounted your own note (in the exercise just given), you did so on the same day that the note was made. The discount period and the time of the note were therefore the same. But you may keep the notes of other people some time before discounting them. The bank will charge you interest from the day you discount the note until it is due.

Thus, if you did not discount the above note until March 6, the bank would charge you interest for the number of days from March 6 to April 16, the date of maturity. How many days in this discount period ?

What would be the discount period if you discounted the note January 23 ? February 16 ? March 20 ?

199. To find the discount period.

Find the maturity of the note.

Find the exact number of days between the date of discount and the date of maturity.

200. To find the maturity. Notes due a certain number of *months* after date fall due on the same day of the month as the day on which they were made, with the following exceptions :

Notes made on the 31st, maturing after a specified number of

months, falling due in a month having only 30 days, mature on the 30th.

Notes made on the 28th, 29th, 30th, or 31st, maturing after a specified number of months, falling due in February, mature on the last day of February.

Examples.

Note Made	Due in	Maturity
Jan. 16, 1915	2 months	March 16, 1915
July 31, 1915	2 months	September 30, 1915
Dec. 31, 1915	2 months	February 29, 1916

Notes due a certain number of *days* after date, mature after the exact number of days has elapsed.

Examples.

	Note Made	Due in	Maturity
	1915		
a.	Feb. 16	30 days	March 18

Since February has only 28 days, the 30 days of the note will be divided as follows:

12 February days (after the 16th)
18 March days

b. March 18 — 30 days — April 17

March has 31 days; 13 left after the 18th; the remaining 17 of the 30 days must be in April.

c. April 17 — 30 days — May 17

April has 30 days; 13 left after the 17th; the remaining 17 of the 30 days will be in May.

From these examples, the following should be clear:

To find the maturity.

Change the days to months; (30 days = 1 month).

Assume that the note will mature on the same day of some following month.

Correct this result by subtracting 1 day for each month of 31 days through which the note runs; and by adding 2 days if the note runs through February. (In case of leap year, add 1 day.)

The following table provides a convenient method for indicating the months for which corrections are to be made.

Month	Number of Month		Days	Correction
January	1	13	31	− 1
February	2	14	28	+ 2 (or + 1)
March	3	15	31	− 1
April	4	16	30	
May	5	17	31	− 1
June	6	18	30	
July	7	19	31	− 1
August	8	20	31	− 1
September	9	21	30	
October	10	22	31	− 1
November	11	23	30	
December	12	24	31	− 1

Examples. 1. Find the maturity of a note dated May 16, 1915, due in 4 months.

Solution. May is the fifth month. Add 4 months. The note will be due in the ninth month, which by the table is September.

Maturity, September 16.

2. Find the maturity of a note of the same date, due in 120 days.

Solution. Call 120 days 4 months.

May is the fifth month. The note is due in the ninth month, shown by the table to be September.

Call the maturity September 16. Correct as follows:

Note runs through May	Subtract 1 day
Note runs through July	Subtract 1 day
Note runs through August	Subtract 1 day
Total	3 days

Therefore the maturity is September 13.

3. Find the maturity of a note dated December 19, 1914, due in 150 days.

Solution. Call 150 days 5 months.

December is the twelfth month. The note is due in the seventeenth month, which, by the table, is May.

Call the maturity May 19, 1915.

Correct as follows:

December	− 1
January	− 1
February	+ 2
March	− 1
April (has 30 da.)	
Note does not run through May.	
Total	− 1

Maturity, May 18, 1915.

4. Find the maturity of a note dated June 16, 1915, due in 70 days.

SOLUTION. Call 70 days 2 months, 10 days.
June 16 plus two months is August 16; plus 10 days is August 26.
To correct:

For June	
For July	− 1
Total	− 1

Maturity, August 25, 1915.

Written Work

Enter the maturity of each note in the proper column:

NOTE DATED	DUE IN 1 MONTH	DUE IN 30 DAYS	DUE IN 2 MONTHS	DUE IN 60 DAYS	DUE IN 150 DAYS	DUE IN 45 DAYS
Sept. 23, 1915						
Oct. 9, 1915						
May 1, 1915						
July 20, 1915						
Dec. 15, 1915						
Feb. 9, 1915						

201. To find the discount period. *Compute the actual number of days between the date of discount and the date of maturity.*

Illustration. A note, dated July 17, due in 4 months, was discounted on August 5. What was the discount period?

SOLUTION. Four months from July 17 is November 17, the date of maturity.
The discount period, therefore, extends from August 5, the date of discount, to November 17, the date of maturity.

To find the number of days between August 5 and November 17:

31	total number of days in August
5	number of August days expired before discounting
26	number of August days in the discount period
30	number of September days in the discount period
31	number of October days in the discount period
17	number of November days in the discount period
104	total number of days in the discount period

Oral Work

After studying the illustration, state a method for finding the number of days in the discount period.

Written Work

1. On a form ruled like the following, enter the date of maturity on the upper line. and the discount period on the lower line.

The notes dated February 12 were discounted April 8.
The notes dated February 17 were discounted March 2.
The notes dated March 23 were discounted April 29.
The notes dated March 17 were discounted April 15.
The notes dated March 30 were discounted May 20.

Dates Notes were Made	Time				
	60 Days	75 Days	100 Days	120 Days	6 Months
Feb. 12, 1915					
Feb. 17, 1915					
Mar. 23, 1915					
Mar. 17, 1915					
Mar. 30, 1915					

2. Complete a table ruled like the following model. Enter the date of maturity on the first line, the discount period on the

second line, the discount on the third line, the proceeds on the fourth line, as illustrated by the first problem in the table.

Date of Paper and Face	Time, 90 Days Discounted April 13	Time, 3 Months Discounted April 11	Time, 120 Days Discounted April 23	Time, 6 Months Discounted May 20
Jan. 16, 1915 $300	April 16 3 days $.15 $299.85	[illegible] [illegible] 2 5' 299.75	May 16 2 3 days 1.15' 298.85	July 16 5 7 days $2.85 297.15
Jan. 30, 1915 $285	[illegible]	[illegible]		
Feb. 19, 1915 $126				

202. Discounting Interest-bearing Notes. If you had in your possession the following note and wished to discount it, you would first add 90 days' interest ($1.50) to the face of the note to determine its *value* at maturity. This note is really Roe's promise to pay you $101.50 at maturity, and the bank will give you as much for it as for a note for $101.50 without interest.

$100.00 Chicago, January 6th 1915

Ninety days after date I promise to pay to the order of (Your name) One hundred and 00/100 Dollars at my office, with interest @ 6%

Value received

No. Due Richard Roe

When discounting interest-bearing paper, compute the interest for the *full time* the note is to run, and add this interest to the face, to determine the *value* at maturity. Find the discount on this value. Subtract the discount from the *value* to determine the proceeds.

Written Work

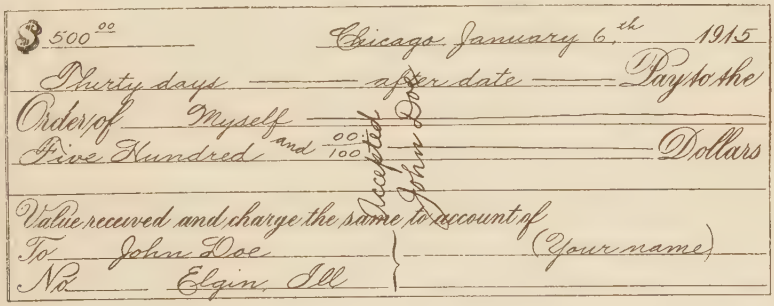
Complete a table ruled like the following model, entering the facts as illustrated in the first problem.

All notes discounted at 6 %.

		Time, 90 Days		Time, 4 Months		Time, 6 Months	
		Without Int.	With Int. 6%	Without Int.	With Int. 6 %	Without Int.	With Int. 5%
Principal, $500.00	Maturity	Sept. 1	Sept. 1				
	Interest		$ 7.50				
Date of Paper, June 3, 1915	Value	$ 500.00	$ 507.50				
	Dis. Period	11 days	11 days				
Date of Discount Aug. 21, 1915	Bank Dis.	$ .92	$ .93				
	Proceeds	$ 499.08	$ 506.57				
Principal, $ 1250.00	Maturity						
	Interest						
Date of Paper, Aug. 6, 1915	Value						
	Dis. Period						
Date of Discount Sept. 18, 1915	Bank Dis.						
	Proceeds						
Principal, $ 47.96	Maturity						
	Interest						
Date of Paper, July 16, 1915	Value						
	Dis. Period						
Date of Discount Aug. 27, 1915	Bank Dis.						
	Proceeds						
Principal, $ 55.00	Maturity						
	Interest						
Date of Paper, Aug. 14, 1915	Value						
	Dis. Period						
Date of Discount Oct. 5, 1915	Bank Dis.						
	Proceeds						

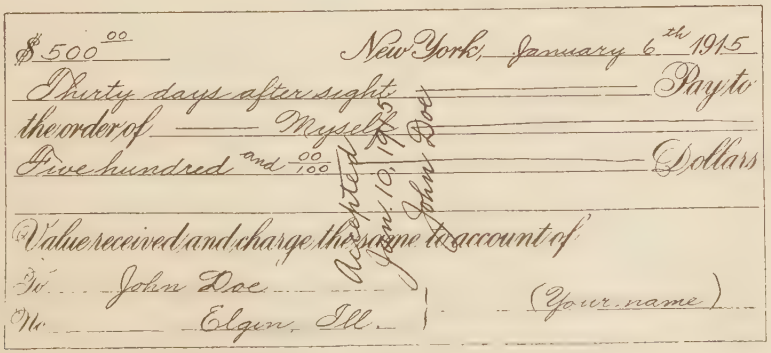
203. Discounting Acceptances. Time drafts were discussed in Chapter XVII. An accepted draft is a written promise of the acceptor to pay money at a definite date, and accepted drafts may be discounted, just as notes may be discounted. In case the person who has accepted the draft does not pay it when due, the person who discounted it will be required to pay it.

The following draft drawn by you on John Doe and accepted by him is just as truly his promise to pay $500.00 at a given date, as a promissory note signed by him would be.


$500 00
Chicago January 6th 1915
Thirty days after date Pay to the
Order of Myself
Five Hundred and 00/100 Dollars
Value received and charge the same to account of
To John Doe
No Elgin, Ill
(Your name)
Accepted Jan 10, 1915 John Doe
NO PROTEST
TAKE THIS OFF BEFORE PRESENTING

This draft could be discounted in exactly the same manner as a note dated January 6, payable 30 days after date.

If the draft were drawn 30 days after sight, instead of after date, the bank discount would be different because the date of maturity would be different.


$500 00
New York, January 6th 1915
Thirty days after sight Pay to
the order of Myself
Five hundred and 00/100 Dollars
Value received and charge the same to account of
To John Doe
No Elgin, Ill
(Your name)
Accepted Jan 10, 1915 John Doe
NO PROTEST TEAR THIS OFF BEFORE PRESENTING

Written Work

1. What is the date of maturity of the draft on page 241 payable 30 days after date?

2. What is the date of maturity of the draft on page 241 payable 30 days after sight?

3. What is the discount period of each draft if discounted on Jan. 14?

4. Find the bank discount and the proceeds of each draft.

Review

Write the notes and drafts called for in the following. Show all acceptances and indorsements.

On October 3, you received from B. A. Anderson, of Batavia, New York, a note dated October 1, due in two months, for $135.00, without interest.

On August 28, you sold to N. M. Davis, of Toledo, Ohio, an invoice of goods amounting to $84.00: terms 1/10; N/30. This invoice was due September 27, but Mr. Davis was unable to pay it. He agreed, however, to accept a thirty-day draft for the amount. You drew the draft on September 30, payable thirty days after date, and sent it to him for acceptance. He accepted it and returned it to you.

On October 5, you sold a bill of goods amounting to $68.00 to J. D. Robinson, of Burlington, Iowa. Terms, 30 day draft less 1%. On October 5, you drew a draft on him, payable 30 days after sight; he accepted the draft on October 7.

On July 10, you received from J. F. Cook of Lime Springs, Iowa, a note made by him on July 8, in your favor, for $500.00, due six months after date, with interest at 6%.

On June 4, you received a note for $600.00 from A. B. Hicks of Chicago. The note was drawn on April 16, by D. F. Fairchild of Omaha, Nebraska, in favor of Hicks, payable nine months after date, without interest, and was transferred to you by full indorsement.

On October 15, you took these notes and drafts to the Dairy State Bank to discount. The bank's discount rate was 6%.

Finding that the proceeds would not be sufficient to meet your needs, you gave the bank your own note, dated October 15, due in 20 days, for $250.00, without interest.

1. Find the proceeds of each of the papers discounted.

Your balance at the bank before discounting this paper was $167.25. You deposited the proceeds of the discounted paper. What was your balance after discounting the paper?

2. On October 25, the bank wished to increase its deposit with its correspondent bank at Chicago. It therefore rediscounted the notes and drafts received from you. How much did the bank receive for the paper, when discounted on October 25 at 6%?

3. Marshall owed Daniels $485.90; on September 23 he gave Daniels the following note, properly indorsed.

$450.00 BRAINERD, IOWA, September 3, 1915.

Sixty days after date, I promise to pay to the order of J. F. Marshall, Four Hundred and Fifty Dollars.

Value received
Without interest. F. S. CROSBY.

How much did Marshall owe Daniels after transferring the note?

CHAPTER XXIV

WAGES AND PAY ROLLS

THERE are several wage systems in use, each of which is designed to encourage employees to produce as much as possible. Several of the most common systems will be discussed in this chapter.

204. The Day or Hour Rate. This system is the one generally used, because it is the simplest. When the day or hour rate is used, the employee's wage depends upon the time he has labored. Work done overtime or on holidays is usually paid for at one and one half times the regular rate. In many factories time clocks are installed. As each employee begins work, he registers the time on his card; when he stops work, he again registers the time. At the end of the week, each employee's card shows his actual hours of labor. A pay roll is prepared showing the total number of hours each employee has worked, his hourly rate, and his total wage.

Written Work

1. Supply the missing facts in the following pay roll.

No.	Name	Hours worked per day						Time			Overtime		Total Wages Earned	Money Advanced	Wages Due
		M	T	W	T	F	S	Hrs	Rate	Amount	Hrs	Amount			
1	Andrew White	9	8	9	9	9	8	52	30	15 60			15 60		15 60
2	August Meier	9	8	11	9	12	[illegible]	[illegible]	20	15 [illegible]	5	[illegible]	[illegible] 32		17 32
3	Edward Ingalls	10	9	11	9	10	9	54	22	?	4	1 32	?	4 —	?
4	James Lane	8	10	9	8	9	9	?	24	?			?		?
5	Oscar Hemmett	9	9	10	11	11	9	?	24	?	?	?	9	1 20	?
6	James Belknap	8	8	9	9	9	8	?	26	?			?		?
7	Fred Calkins	9	10	9	10	11	9	?	30	?	?	?		5 —	?
8	Charles Daves	9	10	10	11	9	10	?	28	?	?	?	?		?
9	Harry McGee	10	9	10	9	10	11	?	25	?	?	?	?	1 75	?

In this factory a full day is 9 hours. Any over hours are paid for at $1\frac{1}{2}$ times the regular rate. Thus, Calkins gets $1\frac{1}{2}$ times regular pay for the overtime on Thursday and Friday, even though he did not work full time on Monday.

2. Rule a pay roll similar to the model, and enter the following data, finding the amount of wages due each employee.

The superintendent supplies the cashier with the following list of the employees, together with their numbers, and the hourly wage rate paid each. A full day in this factory is 9 hours, and overtime is paid for at $1\frac{1}{2}$ times the regular rate.

No.	Employee's Name	Hourly Rate
1.	Fred Smith	$.35
2.	Frank Bailey	.32
3.	Charles Hanchett	.30
4.	Harry Davis	.36
5.	Carl Bell	.38
6.	Lewis Clark	.40
7.	Arthur Helms	.29
8.	Tony Martin	.31
9.	Henry Dickinson	.26
10.	Nathan Wright	.37

During the week ending March 15, time tickets were turned in by the foreman, showing the hours worked by each employee.

Monday: 1, 9; (Workman No. 1, 9 hours); 2, 11; 3, 8; 4, 11; 5, 8; 6, 10; 7, 9; 8, 12; 9, 9; 10, 8.

Tuesday: 1, 9; 2, 11; 3, 9; 4, 9; 5, 10; 6, 9; 7, 9; 8, 9; 10, 9.

Wednesday: 1, 10; 2, 12; 3, 11; 4, 12; 5, 9; 6, 11; 7, 8; 8, 11; 9, 10; 10, 9.

Thursday: 1, 9; 2, 9; 3, 11; 4, 10; 5, 8; 6, 10; 7, 11; 8, 11; 9, 12; 10, 10.

Friday: 1, 5; 2, 9; 3, 11; 4, 10; 5, 9; 6, 9; 7, 9; 8, 9; 9, 10; 10, 8.

Saturday: 1, 9; 2, 11; 3, 11; 4, 10; 5, 11; 6, 9; 7, 12; 8, 9; 9, 4; 10, 9.

The cashier has already advanced Charles Hanchett $3.00; Carl Bell, $2.25; Tony Martin, $3.75; and Nathan Wright, $5.00.

205. Piecework Wage System. The piecework wage system is based on the theory that if the employee is paid for the actual amount of work he accomplishes, he will produce more in a day than if he is paid a straight hour or day wage.

The pay roll prepared under a piecework system is similar to that prepared under the hourly rate system, with the exception that overtime is usually paid for at the same rate that regular time receives, and the basis of the wage is the number of pieces produced and not the number of hours employed.

Written Work

Find the missing facts in the following pay roll.

PIECEWORK PAY ROLL FOR WEEK ENDING JUNE 8, 19—

No.	Name	Operation Number	Number Produced						Total	Rate	Earned	Advanced	Due
			M.	T.	W.	T.	F.	S.					
1	John Sanford	264	16	14	17	13	15	18	93	.14	13 02	2	11 02
2	Irving Danner	178	23	26	22	27	24	25	?	.12	?		?
3	Byron Shepley	361	37	33	34	39	36	30	?	.08	?		?
4	Edward Magee	422	28	27	29	28	26	27	?	.05	?	3 25	?
5	Percival Conrad	267	42	45	41	39	43	20	?	.07	?		?
6	Ernest Anderson	207	24	26	20	23	30	27	?	.09	?	2 60	?

206. The Differential Rate. The differential rate is a modification of the piecework system. A standard day's work is determined, and each employee is expected to produce this standard amount of work. If he produces the standard, he receives a certain rate per piece. If he produces less than the standard, he receives a lower rate per piece. If he produces more than the standard, he receives a higher rate per piece.

This system is based on the theory that the expense of heat, light, rent, power, etc., remains about the same, whether the employees produce a small amount of product, or a large amount. By increasing the amount of output, these expenses are distributed over a larger quantity of manufactured goods, and the cost of making each article is thereby decreased. The saving effected by increasing the output is divided between the owner of the factory and the workmen who, by their skill or industry, increase the output. On the other hand, the employees who, by producing a small quantity, tend to increase the cost per article are paid at a lower rate.

Example. A manufacturer learned by experience that the average workman in his factory could produce ten articles per day, and that 25 cents per article could be paid for the work. He then adopted a differential rate, as follows :

Number of Pieces Produced	Rate per Piece
7	$.20
8	.21
9	.23
10, Standard	.25
11	.26
12	.27

Jones produced the standard quantity on Wednesday, and received $10 \times \$.25 = \$2.50$.

Smith produced 8 pieces, and received $8 \times \$.21 = \$1.68$.

Brown produced 12 pieces, and received $12 \times \$.27 = \$3.24$.

Written Work

The following differential rate applies in a certain factory :

Number of Pieces	Rate
10	$.12
11	.13
12	.14
13	.15
14	.17
15	.19
16	.21
17	.22
18, Standard	.24
19	.25
20	.27
21	.29
22	.31
23	.33
24	.33$\frac{1}{2}$
25	.34

The production sheet for the week ending September 17, 19— was as follows :

No.	Name	Daily Production					
		M.	T.	W.	T.	F.	S.
1	Edward Dye	17	19	18	16	19	18
2	Frank Bartlett	14	15	14	16	15	14
3	Arthur Bassett	19	20	19	21	22	19
4	James Hamilton	22	21	21	23	20	21
5	Den Slocum	12	14	13	15	12	14
6	John Gaylord	13	16	15	16	15	17
7	Sam Bayliss	19	20	19	21	18	17
8	George Bates	19	22	23	20	19	18
9	Wm. Osgood	16	14	12	14	11	16

1. Devise a pay roll blank which will show all necessary facts, and enter each laborer's production and wages. See how good a pay roll form you can devise without asking for suggestions. It should be simple and convenient, and while not being too elaborate, it should give all necessary information.

2. Find the average daily production of each workman. Prepare a graph, showing the standard production by a red line across the sheet, and the workmen's averages by a black curve.

Commission

207. Definitions. An **agent** is a person who transacts business for another.

The **principal** is the one whose business is transacted by the agent.

Principals often pay their agents a commission for their services. The **commission** is usually a per cent of the money value involved in the transaction.

Example. A wholesale store engaged a salesman, agreeing to pay him 8% of his gross sales as commission. How much did

the salesman earn if he sold $1425 worth of goods during a month?

SOLUTION.

$1425	sales
.08	rate of commission
$114.00	commission

Written Work

1. A real estate agent sold a farm belonging to Mr. Robinson for $20,450. The agent received 3 % of the selling price as payment for his services. How much did the agent receive? How much did Mr. Robinson receive after paying his agent?

2. A manufacturer engaged Mr. Seeley as a traveling salesman. Mr. Seeley's contract stipulated that he should receive 7 % of his gross sales as commission. During four weeks, his sales were as follows: $576.25; $498.49; $376.45; $723.50.

What was his commission each week?

3. Mr. Brooks, a merchant, engaged an attorney to collect a number of accounts. The attorney charged for his services 15 % of the amounts collected. What did the attorney receive for collecting $354.75? What was the net amount received by Mr. Brooks?

4. James Hunter bought eggs for a Chicago produce merchant. For his labor in purchasing and hauling them to the station, he received $1\frac{1}{2}$ % of the cost of the eggs. On Tuesday he bought 27 cases, of 30 dozen each, paying 29 cents per dozen for them. How much did Hunter pay for the eggs? How much was his commission? How much did the eggs cost the Chicago merchant, not including the freight?

5. The salesmen of the Wickman Specialty Company receive the following commissions:

On goods sold in Department A, 9 %.
On goods sold in Department B, 13 %.
On goods sold in Department C, $12\frac{1}{2}$ %.
On goods sold in Department D, 15 %.

The following table shows the sales made by five men.

SALESMAN	DEPT. A	DEPT. B	DEPT. C	DEPT. D
C.G. PINES	356.27	473.28	293.75	459.23
F.L. SNOW	523.74	625.90	483.37	824.84
E.D. VICKS	283.57	285.37	295.64	472.84
L.S. BASSETT	923.74	825.36	736.46	462.47
FRANK HOLTZMAN	735.84	375.47	375.47	462.85

Devise a form which will show the above data and enter the following:

Each salesman's commission on goods sold in each department.

The total commissions paid to each salesman.

The total commissions paid for each department.

The total commissions paid to all salesmen.

The total sales in each department.

The total sales of each salesman.

The total sales in all departments.

208. Pay Roll Slips. Laborers are sometimes paid by check, and sometimes by the envelope system. When the latter method is used, each employee receives an envelope containing the exact amount of his wages. A coin sheet is prepared from the pay roll, showing the change needed for each envelope. A pay roll

No.	Name	Wages	Bills				$.50	$.25	$.10	$.05	$.01
			$10.00	$5.00	$2.00	$1.00					
1	James Downs	22 67	2		1		1		1	1	2
2	Francis Canfield	3 87	3			1			1	1	4
3	Arthur Prichard	2 35	2			1		1	1		3
4	Frank Munger	24 00	2	1				1	1		
5	George Selby	29 70	2	1	1	1	1	1	2		1
6	Ezra Colfax	18 15	1	1	1	1			1	1	
		157 20	12	4	4	4	3	3	7	3	10

Coin Sheet

slip, designating the number and denomination of the bills and coins desired, is then filled out and sent to the bank. When the

THE FIRST NATIONAL BANK

CURRENCY FOR PAY ROLL

Depositor Hunter & Colby

	DOLLARS	CENTS	
			$ 50 Bills,
			20
12	120	00	10
4	20	00	5
5	10	00	2
4	4	00	1
3	1	50	Halves,
3		75	Quarters,
7		70	Dimes,
3		15	Nickels,
10		10	Pennies,
	157	20	TOTAL

Pay Roll Slip

currency is delivered a check is made out to the bank for the amount of the pay roll.

Written Work

Rule a coin sheet and enter the number of coins of each denomination necessary to pay each of the employees in the pay roll prepared in the differential wage rate problem on page 247.

Make out a pay roll slip on the First National Bank of your city, to obtain the money to pay these employees.

CHAPTER XXV

POSTAGE RATES

Postage

209. The Rates for Domestic Postage. Domestic mail matter includes all matter deposited in the mails for delivery to points in the United States or its possessions, including Porto Rico, Hawaii, the Philippine Islands, Guam, and the Canal Zone; with certain exceptions, it includes matter sent to Canada, Cuba, Mexico, and the Republic of Panama. The domestic rate applies also to letters, but not to other articles, addressed to Great Britain, Ireland, Newfoundland, and Germany.

CLASSES OF DOMESTIC MAIL

First Class: Written matter: Letters and all mail which is sealed.

Rate: 2 cents for each ounce or fraction thereof. Post cards and postal cards, 1 cent each.

Second Class: Unsealed. Newspapers and periodicals.

Rates: When mailed by the publisher, 1 cent per pound. When mailed by others than the publisher or a news agent, 1 cent for each four ounces or fraction thereof, on each separately addressed copy.

Third Class: Circulars, newspapers, and printed matter (except books) not admitted to the second class.

Rates: 1 cent for each two ounces or fraction thereof. The limit of weight of third-class matter is four pounds.

Fourth Class: Parcel post: All matter not embraced in the first, second, or third classes, and not likely to injure the employees of the postal service, or the mails.

This includes merchandise, farm and factory products, seeds, books, etc.

Rates: Parcels weighing four ounces or less, except books, seeds, plants, etc., 1 cent for each ounce or fraction of an ounce, regardless of the distance.

Parcels weighing eight ounces or less containing books, seeds, cuttings, roots, etc., 1 cent for each two ounces or fraction thereof.

Parcels weighing more than eight ounces containing books, seeds, plants, etc., parcels of miscellaneous printed matter weighing more than four pounds, and all other parcels of fourth-class matter weighing more than four ounces are subject to pound rates, a fraction of a pound being considered a full pound. The pound rates vary with the distance the parcel is to be carried.

210. Units of Area and Zones. For the purpose of determining the various pound rates, the United States is divided into "units of area," each unit being thirty miles square. Each unit is the center of a series of zones:

First Zone: Territory within a radius of 50 miles.

Second Zone: Territory within a radius of 150 miles and beyond 1st zone.

Third Zone: Territory within a radius of 300 miles and beyond 2d zone.

Fourth Zone: Territory within a radius of 600 miles and beyond 3d zone.

Fifth Zone: Territory within a radius of 1000 miles and beyond 4th zone.

Sixth Zone: Territory within a radius of 1400 miles and beyond the 5th zone.

Seventh Zone: Territory within a radius of 1800 miles and beyond the 6th zone.

Eighth Zone: All territory beyond the 7th zone.

211. Size and Weight of Parcels. No package can be sent by parcel post, if its length plus its girth is more than seventy-two

inches. For delivery in the first and second zones, parcels may weigh not more than fifty pounds; for delivery in other zones, they may weigh not more than twenty pounds.

212. Parcel Post Rates. The rates of postage on parcels exceeding 4 ounces in weight are as shown on the opposite page.

The local rate applies to parcels to be delivered from the same office at which they are mailed. The delivery may be made at the office, by city carrier, or by rural free delivery.

Books. Books weighing 4 ounces or less are mailed at the third-class rate of 1 cent for each 2 ounces or fraction thereof. Those weighing more than 4 ounces are mailed under the regular zone rates.

213. Special Regulations. When two classes of mail matter are mailed in the same package, the higher rate is charged on the entire package. For example, inclosing a letter in a parcel of merchandise subjects the whole parcel to the letter rate.

As a general rule the postage on all mail must be prepaid in stamps affixed to the package. Publishers, however, do not always affix stamps to their publications; and the postage on third-class matter, mailed in quantities of not less than 2000 identical pieces, may be paid in money.

Postage stamps are issued in the following denominations: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15, 20, 30, and 50 cents, and 1, 2, and 5 dollars.

Special Delivery. By affixing a "special delivery" stamp (cost, 10 cents), or ten cents' worth of ordinary stamps, in addition to the regular postage, and writing the words "Special Delivery," prompt delivery by special messenger is obtained.

Registry. The registry system provides greater security for valuable mail matter. The registry fee is ten cents. If registered mail is lost, the sender is indemnified up to $50 for first-class, and up to $25 for third-class domestic mail.

Insurance of Parcels. Mail sent by parcel post may be insured against loss upon the payment of a fee of 5 cents for value not exceeding $25, or 10 cents for value not exceeding $50, in addition to the postage. It may not be registered.

Table of Postage Rates on Fourth-class Mail, Effective January 1, 1914.

Weight	First Zone		Second Zone Rate	Third Zone Rate	Fourth Zone Rate	Fifth Zone Rate	Sixth Zone Rate	Seventh Zone Rate	Eighth Zone Rate
	Local Rate	Zone Rate							
1 pound	$0.05	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09	$0.11	$0.12
2 pounds	.06	.06	.06	.08	.11	.14	.17	.21	.24
3 pounds	.06	.07	.07	.10	.15	.20	.25	.31	.36
4 pounds	.07	.08	.08	.12	.19	.26	.33	.41	.48
5 pounds	.07	.09	.09	.14	.23	.32	.41	.51	.60
6 pounds	.08	.10	.10	.16	.27	.38	.49	.61	.72
7 pounds	.08	.11	.11	.18	.31	.44	.57	.71	.84
8 pounds	.09	.12	.12	.20	.35	.50	.65	.81	.96
9 pounds	.09	.13	.13	.22	.39	.56	73	.91	1.08
10 pounds	.10	.14	.14	.24	.43	.62	.81	1.01	1.20
11 pounds	.10	.15	.15	.26	.47	.68	.89	1.11	1.32
12 pounds	.11	.16	.16	.28	.51	.74	.97	1.21	1.44
13 pounds	.11	.17	.17	.30	.55	.80	1.05	1.31	1.56
14 pounds	.12	.18	.18	.32	.59	.86	1.13	1.41	1.68
15 pounds	.12	.19	.19	.34	.63	.92	1.21	1.51	1.80
16 pounds	.13	.20	.20	.36	.67	.98	1.29	1.61	1.92
17 pounds	.13	.21	.21	.38	.71	1.04	1.37	1.71	2.04
18 pounds	.14	.22	.22	.40	.75	1.10	1.45	1.81	2.16
19 pounds	.14	.23	.23	.42	.79	1.16	1.53	1.91	2.28
20 pounds	.15	.24	.24	.44	.83	1.22	1.61	2.01	2.40
25 pounds	.17	.29	.29						
30 pounds	.20	.34	.34						
35 pounds	.22	.39	.39						
40 pounds	.25	.44	.44						
45 pounds	.27	.49	.49						
50 pounds	.30	.54	.54						

Parcels weighing 4 ounces or less are mailable at the rate of 1 cent for each ounce or fraction of an ounce, regardless of distance. Parcels weighing more than 4 ounces are mailable at the pound rates, a fraction of a pound being considered a full pound.

214. Rates for Foreign Postage. The rates of postage applicable to articles for foreign countries are as follows:

	CENTS
Letters for Germany, England, Ireland, Newfoundland, Scotland, and Wales, per ounce	2
Letters for all other countries:	
For the first ounce or fraction of an ounce	5
For each additional ounce or fraction of an ounce	3
Post cards, each	2
Printed matter of all kinds, for each two ounces or fraction of two ounces	1
Commercial papers (business documents without letters):	
For the first ten ounces or less	5
For each additional two ounces	1
Samples of merchandise:	
For the first four ounces	2
For each additional two ounces	1
Parcel post, per pound or fraction	12

Registry fee on all classes of mail, ten cents in addition to the postage.

Written Work

Compute the cost of postage on the following:

Domestic mail.

1. Letter weighing $1\frac{1}{2}$ oz.
2. Book weighing 5 oz.
3. Book weighing 19 oz. to a point in the fourth zone.
4. Newspapers weighing 26 pounds mailed by publisher.
5. Three separately wrapped newspapers mailed by a person not a publisher; weight, 3 oz., 5 oz., and 8 oz.
6. Ten separately addressed printed circulars, each weighing $1\frac{1}{2}$ oz.
7. Package of merchandise weighing 3 oz.
8. Package of seeds weighing 12 oz.
9. Merchandise weighing 16 lb. 7 oz., to a point in the sixth zone.
10. Merchandise weighing 35 pounds, to a point 130 miles distant.
11. Registered letter weighing 7 oz.

12. Books weighing 11 lb. 9 oz., to a point in the fifth zone; package insured. (Ins. fee 5¢.) .79

Foreign mail.

13. Letter to Berlin, weighing 1½ oz.

14. Letter to Paris, weighing 1½ oz.

15. Book to Madrid, weighing 9 oz.

16. Package of merchandise samples weighing 11 oz., to London.

CHAPTER XXVI

PROPERTY INSURANCE

PROPERTY is in danger of destruction from many causes; insurance companies have been organized to assume the risks of destruction, thus relieving the owners from anxiety and loss. Any person having a financial interest in a property may insure himself against its loss or destruction by paying to an insurance company a certain per cent of the value of the property insured.

The person who purchases insurance is called the **Insured.**

The company which assumes the risk is called the **Underwriter.**

The contract between the insured and the underwriter is called a **Policy.**

215. The Value of Insurance.

a. In case of loss the insured receives a certain sum, called an indemnity, which wholly or partially repays his loss.

b. Even though the property is not destroyed, the insured has the security which the insurance gives.

c. Some wholesale houses are willing to sell goods on credit to persons whose property is adequately insured.

216. Risks for which Insurance is Written. Insurance companies are organized to furnish protection against numerous kinds of risks. Some of the most common classes of insurance are :

Fire.
Lightning and Tornado.
Marine, on vessels and cargoes at sea.
Live Stock.
Transportation.
Burglary.
Plate Glass.
Boiler Explosion.
Automobile.
Crop.

The principles underlying all types of property insurance are the same, and for the purposes of this text fire insurance only will be explained. It will be understood that other forms of insurance may be written, settled, or canceled in a similar manner.

217. Process of Insuring. When a person wishes to secure a policy insuring his property, he should first ascertain the value of the property. To do so, it may be necessary to take an inventory, applying to the goods a price at which they would sell in the market at the time of taking the inventory.

After the valuation has been determined, the next thing to decide is the amount of insurance it is desirable to carry. If the property is insured at its total valuation, the owner will bear no loss in case of fire. But since, in most fires, a portion of the property is saved, it is customary to insure only a fraction of the total valuation. Thus, if a stock of goods worth \$10,000.00 is insured at $\frac{3}{4}$ of its value, in case of fire the insurance company will pay all loss up to \$7500.00. If goods to the value of \$2500.00 or more are carried from the building and saved, the merchant will not suffer any loss. If only \$1000.00 worth of goods are saved, the merchant will lose \$1500.00.

It is a matter of choice how much risk the insured wishes to take. If he wishes to be free from all risk, he should insure his property at its full valuation. Insurance companies, however, are rarely willing to insure property at its full value. Business men usually insure from $\frac{1}{2}$ to $\frac{4}{5}$ of the valuation of their property.

In some rural communities where the fire protection is poor, insurance companies sometimes refuse to insure more than $\frac{3}{4}$ of the valuation of property. On the other hand, in cities where the fire protection is good and the probability of total loss is comparatively slight, people are not inclined to insure a large fraction of the valuation. To counteract this tendency, the insurance companies frequently offer a lower rate, or price per hundred dollars of insurance carried, to those who insure 80 % or more of the valuation, than they offer to persons insuring less than 80 %.

218. The Cost of Insurance. Insurance companies charge a certain number of cents or dollars for insuring each $100.00 worth of property. Thus, if the insurance rate were 60 cents per hundred dollars of insurance carried, and you insured a building worth $5000.00 at its full value, the insurance cost would be computed as follows:

$.60 the rate per $100 insurance
50 the number of hundred dollars of insurance purchased
$30.00 the premium, or cost of the insurance per year

219. To find the premium:

Multiply the valuation of the property by the fraction of the value insured.

Point off two places in this product, to determine the number of hundred dollars of insurance purchased.

Multiply by the rate per $100.

Example. What is the premium for insuring, at 80% of its value, a building worth $10,000.00, if the rate is 90 cents per year?

SOLUTION. $10,000.00 value of the property
.80 fraction of property to be insured
$8,000.00 insured value

$.90 rate per hundred
80 number of hundred dollars of insurance carried
$72.00 premium

220. Insurance Rates. The insurance rate per hundred dollars per year varies from $.30 on brick dwellings in cities, where the fire protection is good, to $7 or more on very dangerous risks, such as garages storing gasoline, and buildings located in districts where the fire protection is poor.

Thus it is seen that the rate depends upon the hazard, or the danger of the property burning. This hazard is determined by four things, which are always taken into account in fixing the rate on different pieces of property. They are:

a. The **Construction** of the building. If it is built of wood a higher rate is charged than if it is built of brick, or other slow-burning material. If it has many stories, so that a fire starting

near the roof will be difficult to extinguish, a higher rate will be charged.

b. Its **Occupancy.** If the building is occupied by some hazardous business, such as a wood-working mill in which wood shavings accumulate, or a clothes-cleaning establishment containing dangerous chemicals, a higher rate will be charged than if the business caused no particular hazard.

c. Its **Exposure.** By "exposure" is meant: first, the distance from other buildings from which it might take fire; and second, any hazard which might arise from its being in the neighborhood of a dangerous building.

d. Its **Protection.** The rate will be lower if the building is close to a fire hydrant, in a city with an efficient fire department, or if the building is equipped with automatic sprinklers which are opened by the heat from a fire. If a night watchman is engaged, this also may reduce the rate.

The rate depends also upon the length of time for which the insurance is purchased.

221. Typical Rates. The following table will illustrate the rates which apply on different classes of property. On business houses and factories the rate is fixed after an inspection of the property made by a representative of the insurance company. The insurance companies of a state frequently join in organizing an "inspection bureau" which fixes the rates.

The rates on city property will be lower than those which apply generally throughout the country, because of the more adequate fire protection of a large city.

A City Insurance Tariff

DWELLINGS

	One Year, per $100
Brick, stone, tile, cement-block, or concrete dwellings and contents	$0.30
Single frame dwellings and contents, detached not less than 50 feet in all directions	.50
Single frame dwellings and contents, detached not less than 25 feet in all directions	.60
Single frame dwellings and contents, detached 50 feet on one side	.60

	One Year, per $100
All other frame dwellings and contents, not less than	.75
Brick-veneered or tile-veneered dwellings and contents	.40
Dwellings plastered outside, with tile or other non-combustible roof, and contents	.40

Dwellings plastered outside, with shingle or other combustible roof, rate same as frame dwellings.

Dwellings in part brick and frame rate as frame, subject to survey and rating by the board.

Streets, without reference to their width, to be considered as cutting off charge for exposure.

BRICK BUILDINGS AND CONTENTS, OCCUPIED FOR APARTMENT HOUSES OR FLATS

	One Year, per $100
Three stories in height	$0.40
Four stories in height and 50 feet or less wide	.50
Five stories in height and 50 feet or less wide	.60
Six stories or more in height and 50 feet or less wide	.75

BRICK BUILDINGS (OTHER THAN APARTMENT HOUSES, CHURCHES, SCHOOLS, FLATS AND WOODWORKING RISKS) IN PROCESS OF CONSTRUCTION

	One Year, per $100
Two stories or less in height	$0.40
Three stories in height	.50
Four stories in height	.60
Five stories in height	.75

Over five stories in height, add 25 cents for each story.

SCHOOLHOUSES

	One Year, per $100
Brick or stone, with metal, slate, or composition roof and contents	$0.60
Brick or stone, with shingle roof, and contents	.75
Frame, and contents	1.00

CHURCHES

	One Year, per $100
Brick or stone, and contents	$0.75
Frame, and contents	1.00

TERM RATES

When policies are written for several years, the following will apply:

2 years, $1\frac{1}{2}$ annual rates.
3 years, 2 annual rates.
4 years, $2\frac{1}{2}$ annual rates.
5 years, 3 annual rates.

FARM PROPERTY

	One Year, per $100	Three Years, per $100	Five Years, per $100
Dwellings, barns, outbuildings, and contents, when written under same policy	$0.50	$1.00	$1.50
When farm barns and contents are written without the dwellings	.75	1.50	2.25

Written Work

Compute the premiums on the following fire insurance policies. The proper rates will be found by a study of the preceding tariffs.

ONE-YEAR POLICIES — CITY PROPERTY

1. Single frame building, value $4000. No buildings nearer than 60 feet. Insured at 80 % of its valuation.

2. Single frame dwelling, value $5400. House on north, 70 feet away; store on south, 25 feet away. Insured at full value.

3. Brick dwelling, value $6800. Insured at 75 % of its value. Nearest building, 50 feet.

4. Dwelling, plastered outside; shingle roof. Value, $4200. Insured for $3600. No building nearer than 35 feet.

5. Five-story apartment house, 48 feet wide. Value, $36,000. Insured at $\frac{3}{4}$ of value.

6. Four-story brick store in process of construction. Value, $3900. Insured at 80 % of its valuation.

TERM POLICIES — CITY PROPERTY

7. Three-year policy on brick schoolhouse, with tile roof. Value $40,000. Insured at full value.

8. Five-year policy on frame church. Value, $7500. Insured at full value.

Policies on Household Goods — City

9. Three-year policy of $1000 on household goods in frame dwelling. Building on north, 10 feet; building on south, 60 feet.

10. One-year policy of $500 on household goods in brick-veneered building.

Farm Policies

11. One-year policy of $8500, covering dwelling and barns.

12. Five-year policy of $3000 on barn and silo.

CHAPTER XXVII

TAXATION

222. Purpose of Taxation. The national government requires money to support the army and navy, to pay the salaries of government employees, to pay pensions, and to finance other activities carried on by the nation. During a recent year, Congress appropriated $1,098,678,788 for the annual budget.

The state governments require money for the expense of their officers, and to support their various institutions, schools, universities, asylums, and penitentiaries.

The counties require money for the building of bridges, the trial of criminal cases, the salaries of officers, the relief of the poor, etc.

Cities must pay for police and fire protection, care of streets, etc.

School districts contribute to the support of the public schools.

The money required for all these expenses is raised by taxes, licenses, fees, assessments, and fines.

223. A **tax** is a sum of money levied by the proper officers of a government to defray governmental expenses.

The funds of the national government are raised largely in three ways:

Customs Duties on Imports;
Internal Revenue;
Income Tax.

The funds of the state and local governments are raised largely by direct taxes on real estate and personal property.

State and Local Taxes

224. Taxable Property. The amount of tax paid by any individual to state and local governments depends upon the value of the property which he owns and the tax rate.

Real estate consists of land and buildings.

Personal property consists of movable property, such as merchandise, furniture, machinery, live stock, cash, notes, stocks, bonds, and mortgages.

225. Determining the Assessed Value of Property. The value of each person's taxable property is determined by the assessor. The **assessed value** is usually a fractional part of the real value.

For example, in a certain state, property is assessed at $\frac{1}{3}$ of its real value. Jones owns a farm which the assessor considers to have a real value of $30,000. He therefore assesses it at $10,000.

Although the law gives the assessor the power to determine the taxable value of property, property owners usually have the privilege of appearing before a Board of Equalization to prove a claim that their property has been assessed at too high a valuation. The Board of Equalization judges between the values fixed on property by the assessor and by the owner.

226. The Tax Rate. The tax rate may be expressed in several ways, the most common of which are:

A per cent;

A certain number of mills on each dollar of assessed valuation;

A certain number of dollars or cents on each hundred dollars of assessed valuation.

The following tax rates are equivalent:

1.6 %;
16 mills (on the dollar);
$1.60 (on each hundred dollars).

227. To find the amount of tax. To find the amount of tax to be paid by any property owner:

Multiply the assessed value of the property by the tax rate.

(*a*) When the rate is stated as a per cent.

Example. Bennet's property is assessed at $3000; the tax rate is 1.6 %.

SOLUTION.

$3000 assessed valuation
.016 tax rate
$48.00 tax

(*b*) When the rate is stated as a certain number of mills on the dollar.

Example. Taylor's property is assessed at $3800. The rate is 24 mills.

SOLUTION.

$3800 assessed valuation
.024 tax rate in mills
$91.20 tax

(*c*) When the rate is stated as a certain number of dollars on each hundred dollars of assessed value.

Example. Finch's property is assessed at $5470. The tax rate is $1.95.

SOLUTION.

$1.95 the rate per hundred dollars
54.70 the number of hundreds of dollars assessed value
$106.67 the tax

Oral Work

1. State each of the following tax rates in two other ways:

3.6 % 27 mills $2.83

2. A certain state assesses property at $\frac{2}{5}$ of its real value. What is the assessed value of a house worth $5000? Of a factory worth $15,000? Of a building worth $4000?

3. Dickinson has a house assessed at $2000. The rate is 2.5 %. What is his tax?

4. Barnes has a store building assessed at $3000 and a house assessed at $2000. The tax rate is 30 mills. What is his tax?

5. If property is assessed at $\frac{1}{3}$ of its real value, what is the assessed value of a city lot worth $1200? How much tax will the owner pay if the tax rate is 22 mills?

6. Property valued at $6000 is assessed at $\frac{1}{5}$ of its value and taxed at a rate of $2.50. What tax does the owner pay?

7. Three men living in different cities were comparing their tax rates. A's rate was $3.56; B's rate was 28 mills; C's rate was 3.14 %. Which had the largest rate? How much tax should each pay on an assessed value of $2000?

8. Bailey owns property worth $ 15,000 and is taxed $4.50 per hundred dollars on $\frac{1}{5}$ of the real value. Osborne owns property worth $ 15,000 and is taxed 32 mills on $\frac{1}{3}$ the real value. Which pays the larger tax? How much does each pay?

9. Powell pays 3% on $\frac{1}{3}$ of the real value of his property. What per cent of the value of his property does he pay annually as taxes?

10. A loaned money at 6% and took a mortgage. Since a mortgage is personal property, it is taxable. The mortgage was taxed at a rate of $4 on $\frac{1}{5}$ the real value. What per cent of the value of the mortgage did A pay annually as taxes? What was his per cent of net income on the loan?

Written Work

1. Complete the following table:

Real Value of Property	Fraction of Value Assessed	Assessed Value	Rate	Tax
$16,250	$\frac{1}{5}$		$5.42	
7,920	$\frac{1}{3}$		3.18%	
22,000	$\frac{2}{5}$		$2.89	
960	$\frac{1}{5}$		64 mills	
4,000	$\frac{1}{3}$		2.92%	

2. Hill rents his house for $30.00 per month. The house cost him $3850.00. What is the per cent of net profit on his investment, after paying the following annual expense?

Insurance, $15.40.

Repairs, $60.00.

Taxes: Property assessed at $\frac{1}{5}$ its value and taxed at $6.23 per hundred dollars.

CHAPTER XXVIII

BUYING EXPENSES; SELLING EXPENSES; NET PROFIT

228. Buying Expenses include all the costs of buying goods, such as freight and drayage, customs duties on goods imported, commissions of purchasing agents, insurance, keeping goods in warehouses, and placing them on the shelves of the salesroom. Buying expenses are added to the original purchase price of goods to determine the total cost. Thus:

Prime Cost of Merchandise is the original or purchase price;

Total Cost of Merchandise is the prime cost plus the buying expenses.

Example. A retail furniture store purchases goods during the year, amounting to $36,450.00. This is the prime cost. The freight, drayage, rent of warehouse, labor of warehouse employees, and other costs of placing the goods in the salesroom are $2187.00. What is the total cost of the merchandise purchased?

SOLUTION.

$36,450.00	Prime cost
2,187.00	Buying expenses
$38,637.00	Total cost

229. Marking Cost of Goods. When the cost price is marked on goods, the total cost, and *not* the prime cost, should be used. This is easily understood if we consider a case in which a merchant is taking an inventory of his stock. He should value his stock at the prime cost plus buying expenses necessary to get the goods on his shelves.

Some merchants add a certain per cent to the prime cost to cover the buying expenses. The records of previous years are used as a basis to determine the per cent to be added.

In the illustration above, the prime cost is $36,450.00, and the buying expenses $2187.00, or 6% of the prime cost. The furniture dealer might say, "Next year's buying expenses will probably

be about the same as they were this year, and I will therefore add 6% to the prime cost to cover them."

A safer method, however, is to base the computation on the average expenses of several years.

Year	Prime Cost	Buying Expense
1913	$35,275.00	$1840.13
1914	41,391.00	2069.55
1915	62,387.00	3618.45
	$139,053.00	$7528.13

$$\$7528.13 \div \$139,053.00 = 5.41^{+}\%.$$

For convenience, 5.4% would probably be adopted as the per cent of buying expenses.

Illustration. An article cost $8. At what price is it marked to cover buying expense?

Solution.

$8	Prime cost
.055	Per cent of buying expenses
$ .44	Buying expenses
$8.00	Prime cost
.44	Buying expenses
$8.44	Total marked cost

Written Work

The prime cost of a merchant's purchases and the buying expenses for several years are given below. Find the per cent of buying expenses for each year and the average for the entire interval.

	Year	Purchases	Buying Expenses
1.	1911	$ 9,426.90	$643.92
	1912	12,318.26	812.29
	1913	14,126.29	830.12
	1914	11,341.72	690.19
	1915	14,985.29	752.29
	Year	Purchases	Buying Expenses
2.	1913	$39,286.26	$2326.80
	1914	47,562.83	3146.29
	1915	59,286.28	4210.08

3. The prime cost of merchandise in a certain store is increased 8% to include buying expenses; the selling price is determined

by adding 9% to the total cost. Mark the total cost (using "blacksmith" as the key and "g" as a repeater) and the selling price (in figures) of articles, the prime cost of which was as follows:

Stock No.	Prime Cost
A 326	$2.19
A 394	6.25
F 246	.90
A 385	2.78

4. The manager of one of the departments in a large store adds 6% to the prime cost to cover the buying expenses. He receives quotations of prices from manufacturers who offer different discounts. He prepares a table showing the total cost after deducting the discount offered and adding the 6% buying expenses. The following is a similar table which you will complete:

	.00			.25			.50			.75		
$	Less 2% Plus 6%	Less 5% Plus 6%	−7% +6%	−2% +6%	−5% +6%	−7% +6%	−2% +6%	−5% +6%	−7% +6%	−2% +6%	−5% +6%	−7% +6%
0				.2597	.25175	.24645						
1	1.0388	1.007	.9858									
2												
3												
4												
5												
6												
7												
8												
9												
10												

5. By referring to the table, find the total cost, including buying expenses, of an article quoted at $2.25 less 5%; $3 less 2%.

Refer to the table, and state

6. Which is the better price, and how much better: $1.75 each, less 2%, or $2 each, less 7%? How much will be saved by purchasing one gross at the cheaper quotation?

230. Selling Expenses. After the goods have been placed on sale, all further costs, such as rent of store, salaries of clerks, advertising, heat and light, delivery, store supplies, insurance and taxes, depreciation and shrinkage, bad debts, and general expenses, are considered as selling expenses.

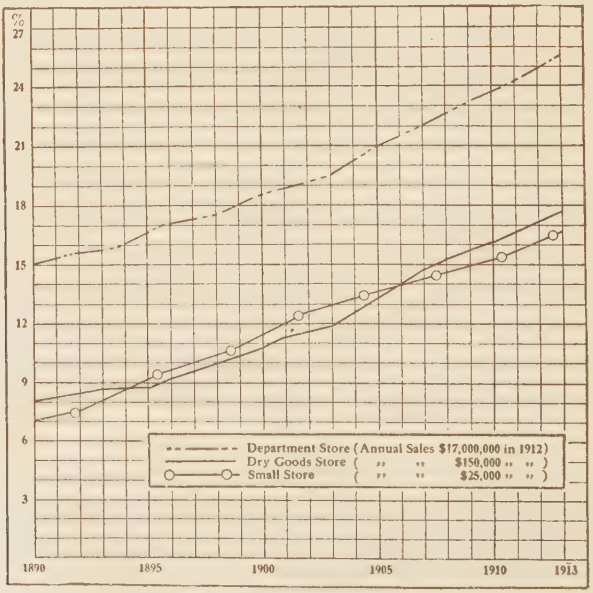
231. Finding the per cent of selling expenses. The per cent of selling expenses is a matter of valuable information to a merchant. It is determined by the formula:

Selling Expenses ÷ Gross Sales = % of Selling Expenses.

Example. A merchant's gross sales during one year are $124,265; the selling expenses for the same year are $29,078.01. The selling expenses are what per cent of the gross sales?

Solution. $29,078.01 ÷ $124,265.00 = 23.4 %.

232. Increase of Selling Expenses. During recent years, as shown by the graph reprinted from *System, a Magazine of Business*, selling expenses, such as advertising, delivery, etc., have greatly increased.



Oral Work

Trace the rise in the per cent of selling expenses in each business, stating what per cent the selling expenses in each business are of the gross sales in each year.

Written Work

The following statistics are reprinted from *System*, and show the result of an investigation made by that magazine to determine *average* or *standard* per cents of selling expenses in various businesses.

Find what per cent each item of expense is of the total sales.

Find what per cent the total selling expenses are of the sales.

	Dry Goods Store	General Store	Shoe Store
	Gross Sales, $50,000.00	Gross Sales, $50,000.00	Gross Sales, $25,000.00
Rent	$1,550.00	$1,154.85	$ 778.13
Salaries	4,800.00	4,067.09	2,786.21
Advertising	750.00	351.48	376.51
Heat and light	200.00	251.04	225.91
Delivery	450.00	954.01	75.30
Supplies	200.00	150.63	100.40
Insurance and taxes	550.00	200.84	301.21
General expense	2,200.00	150.63	1,029.14
Depreciation and shrinkage	700.00	301.27	150.61
Bad debts	150.00	150.63	25.10

	Vehicle Store	Hardware Store	Furniture Store
	Gross Sales, $57,000.00	Gross Sales, $45,000.00	Gross Sales, $100,000.00
Rent	$1,094.63	$1,305.61	$3,50[illegible].36
Salaries	5,818.81	5,042.35	8,317.60
Advertising	633.73	405.20	2,906.15
Heat and light	345.67	225.10	801.70
Delivery	518.51	270.13	1,102.33
Supplies	230.45	180.08	501.06
Insurance and taxes	460.90	495.23	1,402.97
General expense	230.45	360.17	1,803.82
Depreciation and shrinkage	403.28	225.10	1,904.03
Bad debts	115.22	180.08	1,202.54

	Clothing Store	Drug Store	Jewelry Store
	Gross Sales, $60,000.00	Gross Sales, $20,000.00	Gross Sales, $30,000.00
Rent	$1,322.24	$ 924.65	$1,080.40
Salaries	5,469.29	2,191.01	3,361.23
Advertising	2,043.47	522.64	1,050.38
Heat and light	180.31	160.81	180.07
Delivery	360.61	80.40	30.01
Supplies	120.20	60.30	270.10
Insurance and taxes	661.12	281.40	540.20
General expense	1,681.84	482.42	630.23
Depreciation and shrinkage	721.22	100.50	360.13
Bad debts	240.41	40.20	90.03

Save your results for use in the problems which follow.

233. Efficient Management to Determine Economies and "Leaks." When a merchant has a reliable standard with which to compare the various per cents of selling expenses in his business, he is able to determine the items of expense which are too large.

Written Work

A dry goods store has gross sales during one year amounting to $40,000.00. The expenses are as follows:

Rent	$1200.00	Supplies	$165.00
Salaries	3240.00	Insurance and taxes	180.00
Advertising	250.00	General expense	155.00
Heat and light	225.00	Depreciation and shrinkage	120.00
Delivery	1135.00	Bad debts	42.00

1. Each item of expense is what per cent of the gross sales?

2. The total selling expenses are what per cent of the sales?

3. Accepting the per cents found in the dry goods store problem on page 273 as a standard, determine which expenses are larger than the standard, and which are smaller.

4. Complete the following table:

Profits in 12 Departments

Dept.	Stock, Jan. 1, 1915	Purchases During 1915	Total Cost for 1915	Stock, Dec. 31, 1915	Cost of Goods Sold	Sales, 1915	Gross Profits	%	Expenses	Net Profits	%	Net Loss	%
A	$15,223 40	$118,427 56		$17,327 72		$162,341 72			$26,514 95				
B	21,714 32	168,427 92		25,314 71		184,231 76			25,601 51				
C	26,361 71	176,204 91		25,747 39		197,432 65			21,430 13				
D	13,247 91	112,347 29		15,261 03		120,261 32			29,675 55				
E	20,403 71	150,265 97		20,374 76		174,231 61			14,512 53				
F	27,349 61	160,231 39		28,417 19		174,628 32			16,669 17				
G	25,732 92	237,511 86		27,961 31		284,162 39			13,635 20				
H	21,113 65	107,221 13		22,213 47		135,261 34			15,509 01				
I	23,416 47	121,376 41		24,316 19		142,221 32			13,459 85				
J	30,265 45	164,213 96		29,262 47		174,227 42			11,581 19				
K	18,462 31	174,216 91		19,417 13		202,346 21			9,420 57				
L	15,271 34	121,216 09		16,237 41		161,213 47			14,748 47				
Total													

Per cent of Gross Profit is found by dividing the Gross Profit by the Sales.
Per cent of Net Profit is found by dividing the Net Profit by the Sales.
Per cent of Net Loss is found by dividing the Net Loss by the Sales.

5. Plot a curve in black ink to show the per cent of gross earnings in each department. Plot a curve on the same paper in red ink to show the per cent of net profit or net loss in each department.

Suggestion. Draw a line horizontally across the paper representing 0 %; scale the paper to show per cents of gain above this line, and per cents of loss below the line.

CHAPTER XXIX

FARM RECORDS

234. The Cash Book. One of the most important records for a farmer is the cash book, in which he can tabulate the various sources of his income and the amounts of his various expenditures.

The following are three of the forms which may be used for this purpose:

A

1915			Receipts	Expenditures
Oct.	1	Balance	123 75	
	1	15 bu. apples @ .70	10 50	
	1	12 doz. eggs @ .30	3 60	
	3	Groceries		5 95
	5	Lumber		26 25

B

1915			Receipts		1915			Expenditures	
Oct.	1	Balance	123	75	Oct.	1	Groceries	5	95
	1	15 bu. apples @ .70	10	50		5	Lumber	26	25
	1	12 doz. eggs @ .30	3	60					

C

1915		Item	Receipts									
			Dairy		Poultry		Crops		General			
May	1	2 cows to Owen	125	00								
	2	10 doz. eggs @ .28			2	80						
	3	4 T. hay					60	00				

EXPENDITURES

1915			DAIRY	POULTRY	CROPS	HOUSEHOLD	GENERAL
May	1	500 # bran	8 00				
	2	10 bu. seed oats			10 00		
	3	Groceries				3 60	
	4	Oyster shell		1 50			
	5	Cement					8 00

Written Work

Rule a form for a cash book similar to the last illustration.

Include an extra column in both "Receipts" and "Expenditures" labeled "Stock" and enter the following facts:

March 1, Balance, $280.00.
1, Sold for cash 1 carload hogs, $1080.00.
1, Bought 2 cows, $45.00 and $58.00.
2, Bought 1 set double work harness, $21.00.
3, Bought 1 work horse, $160.00.
4, Sold 30 doz. eggs at 25 cents.
5, Sold 8 T. timothy hay at $14.00 per ton.
6, Sold 1 wagon load oats to elevator, 95 bu., at 46 cents.
6, Bought groceries, $7.95.
8, Sold one second-hand riding plow, $15.00.
9, Bought 20 sacks cement at $.50.
10, Bought 90 bu. seed oats at 48 cents.
11, Bought 500 # middling for hogs at $30.00 per ton.
12, Bought 2 riding plows at $55.00 each.
15, Received milk check for 30 da., $26.47.
16, Bought groceries, $6.83.
17, Bought wheat screening for chickens, $2.50.
20, Sold 2 steer calves at $9.00 and $12.00.
22, Sold 18 young roosters, 57 # at 14 ¢ per pound.
24, Rented a three-horse team to a neighbor for $2\frac{1}{2}$ da. at $3.75 per day.
26, Bought 10 T. rock phosphate at $3.00 per ton.

28, Sold 28 doz. eggs at 27 cents.

28, Bought 1 pair shoes, $2.85; suit of clothes, $11.00.

Find the total receipts and expenditures for each column, and the balance at the end of the month.

235. Farm Profits. The annual increase or decrease of the farmer's wealth may be determined by taking an inventory each year, and comparing it with the inventory of the preceding year. The inventory should be classified to show the capital invested in land, building, live stock, machinery, and other property. An inventory, taken from Farmers' Bulletin 511, U. S. Department of Agriculture, is reproduced on pages 280 and 281.

236. What is Farm Profit? The annual increase shown by the two inventories is not the farmer's net gain for the year. The real annual profit is found by the following method:

Increase shown by inventories, plus living supplied by farm to family, plus interest paid on indebtedness, minus interest on capital invested, minus wages of farmer and family, equals actual net profit.

Example. Let us suppose the inventories show an increase for the year of $2000. The farm has also supplied a residence and provisions for the farmer's family. If the house rent is estimated at $120 and the supplies are valued at $900, the total, $1020, should be credited to the farm. Adding $80 interest paid on the farm mortgage, we have the following:

$2000	Increase of inventory
1020	Household expenses
80	Interest paid
$3100	Produced by farm

The farmer has $10,000 invested in the farm. If this money were loaned at 5 % interest, it would produce $500 interest annually. $500 interest must therefore be subtracted.

If the farmer had worked for some one else, he might have earned wages of, say, $700. This amount must also be subtracted in order to determine real profit.

Thus, we have

$3100 − $500 (interest) − $700 (wages) = $1900, profit.

Sample farm inventory: Farm of_____

Property	April 1, 1911				April 1, 1912			
	No.	Rate	Valuation		No.	Rate	Valuation	
REAL ESTATE								
Farm of 180 acres (155 tillable), including buildings (dwelling $1,600, barns $1,800, other buildings $600), fences, and other improvements				$17,500.00				$18,500.00
LIVE STOCK								
Dairy cattle:								
Cows, dry and in milk	24	$50.00	$ 1,200.00		26	$50.00	$ 1,300.00	
Bull	1		50.00		1		45.00	
Calves	6	14.00	84.00		8	15.00	120.00	
Two-year olds	4	28.00	112.00		6	20.00	120.00	
Total value of dairy cattle				1,446.00				1,585.00
Hogs:								
Brood sows	2	22.00	44.00		2	21.00	42.00	
Pigs	8	4.00	32.00		6	3.00	18.00	
Total value of hogs				76.00				60.00
Horses:								
Horse, Jim, 7 years old	1		200.00		1		180.00	
Team, Nell and Bess, 5 and 6 years old	1		425.00		1		425.00	
Team, Jack and Prince, 6 and 7 years old	1		400.00		1		400.00	
Colt, 1 year old	1		75.00		1		145.00	
Total value of horses				1,100.00				1,150.00
Poultry:								
Hens	160	.60	96.00		125	.60	75.00	
Roosters	5	1.00	5.00		4	1.00	4.00	
Turkeys	2	3.00	6.00		3	3.00	9.00	
Total value of poultry				107.00				88.00
Total value of live stock				2,729.00				2,883.00
MACHINERY AND TOOLS								
Grain binder	1		90.00		1		82.00	
Sulky plows	2	45.00	90.00		2	41.00	82.00	
Disk harrow	2	28.00	56.00		2	25.00	50.00	
Mower	1		85.00		1		30.00	
Hay rake	1		20.00		1		19.00	
(List all items of farm machines, wagons, harness, and small tools.)								
Total investment in machinery and tools (not all listed here)				475.00				461.00

Sample farm inventory: Farm of ——— —(Continued)

Property	April 1, 1911				April 1, 1912			
	No.	Rate	Valuation		No.	Rate	Valuation	
FEED AND SUPPLIES								
Farm products:								
Corn ..... bushels..	80	.60	48.00		125	.60	75.00	
Oats ..... do....	200	.42	84.00		90	.50	45.00	
Potatoes ..... do....	40	.75	30.00		80	.60	48.00	
Hay, timothy ..... tons ..	10	16.00	160.00		20	15.00	300.00	
Hay, mixed ..... do....	5	12.00	60.00		4	12.00	48.00	
Silage ..... do....	40	4.00	160.00		40	4.00	160.00	
Bran ..... do....	0½	.....	15.00		....	.....	.....	
Mixed feed ..... do....	1	.....	31.00		2½	30.00	75.00	
Seed oats ..... bushels..	30	.80	24.00		35	.80	28.00	
Seed potatoes ..... do....	45	.80	36.00		50	1.00	50.00	
Seed corn ..... do....	3	2.00	6.00		3	2.00	6.00	
Cement ..... sacks..	4	.50	2.00		....	.....	.....	
Twine ..... pounds..	20	.10	2.00		10	.10	1.00	
Total value of feed and supplies	....	.....	.....	658.00	....	.....	.....	836.00
BILLS RECEIVABLE								
J. A. Brown, hay ..... tons..	2	13.00	26.00					
R. S. Jones, potatoes .. bushels..	40	.50	20.00					
Total	....	.....	.....	46.00				
CASH								
On hand	....	.....	90.00		....	.....	210.00	
In bank	....	.....	580.00		....	.....	1,938.00	
Total	....	.....	.....	670.00	....	.....	.....	2,148.00
BILLS PAYABLE								
Farm mortgage	....	.....	.....	2,000.00	....	.....	.....	1,500.00
SUMMARY								
Real estate	....	.....	13,500.00		....	.....	13,500.00	
Live stock	....	.....	2,729.00		....	.....	2,883.00	
Machinery and tools	....	.....	475.00		....	.....	461.00	
Feed and supplies	....	.....	658.00		....	.....	836.00	
Bills receivable	....	.....	46.00		....	.....	.....	
Cash on hand and in bank	....	.....	670.00		....	.....	2,148.00	
Total investment	....	.....	.....	18,078.00	....	.....	.....	19,828.00
Bills payable	....	.....	.....	2,000.00	....	.....	.....	1,500.00
Net worth	....	.....	.....	16,078.00	....	.....	.....	18,328.00
Increase in inventory $2,250.								

Real profit may be determined by the preparation of a statement similar to the following:

Item	March 1, 1914	March 1, 1915
Resources		
Real estate	$15,000	$15,000
Live stock	3,160	3,590
Machinery and tools	530	575
Feed and supplies	860	735
Cash	140	275
Bills receivable		50
Accounts receivable	93	126
Total resources	19,783	20,351
Liabilities		
Accounts payable	135 60	215 20
Mortgage	1,000	1,000
Total liabilities	1,135 60	1,215 20
Present worth	18,647 40	19,135 80
Increase in net worth		488 40
Add		
Interest paid on mortgage 5% on $1000		50
Personal and household expense paid in cash		750
Supplies furnished by farm for household		363
Rent of farmhouse		120
Gross farm gain		1,771 40
Deduct		
Interest on net investment at 5%	932 37	
Labor of owner and family (estimated)	600	
Total deduction		
True net gain		1,532 37
		239 03

Written Work

From the following facts prepare a statement similar to the preceding illustration.

Condition of farmer's affairs on March 1, 1914.

He owns a farm of 166 acres, worth $225 per acre, on which he has given a mortgage of $6000, bearing 5% interest. Cash on hand and in bank, $345.85. His live stock is worth $5280; his

poultry $286; his machinery $1250; unsold crops $900. He has just sold a car of hogs to a local stock buyer, who owes him $975 for them. He also has on hand fertilizer, lumber, and other supplies worth $387.50. He owes sundry accounts, amounting to $236.25.

During the year between March 1, 1914, and March 1, 1915, he paid the interest on his mortgage, and also paid $1000 on the principal of the mortgage. He paid by cash and by butter, eggs, and other produce, for household expenses, $596.50. He estimated the rent of his house at $15 per month; the produce of the farm consumed by the family $645; and his wages $600. He paid taxes and insurance, $275.

Compute interest on net investment at the beginning of the year at 5 %.

On March 1, 1915, he had, cash $167.20; live stock, $6140; poultry, $270; machinery, same as previous year, less 10 % depreciation, plus $165 worth of new machinery; crops on hand, $650; miscellaneous supplies, $295. He owes personal accounts amounting to $193.70 and the unpaid balance of the mortgage.

237. Finding the Profits by Crops. In order to determine the profit from any particular crop, the farmer must know:

a. The value of the crop produced;

b. The cost of producing the crop, including seed, fertilizer, labor, and a share of the taxes, interest, and general expenses.

238. Determining the Expenses. The cost of seed and fertilizer may be determined with comparative ease, because measurable quantities are put on each field.

Taxes, interest, and general expenses are prorated among the different crops in the ratio of the land occupied.

Example. A farmer who has 40 acres of hay, 60 acres of corn, and 30 acres of oats under cultivation, finds that his total interest, taxes, and expenses are $1200. How should this be prorated?

SOLUTION.

$1200 ÷ 130 = $9.23	Charge per acre.
40 × $9.23 = $369.20	Share for hay crop.
60 × $9.23 = $553.80	Share for corn crop.
30 × $9.23 = $276.90	Share for oat crop.

Labor of men and horses must be determined by records of the actual labor spent on each crop.

Accurate labor records may be obtained by a daily time memorandum similar to the following illustration:

Form A.

Regular Worker's Daily Time Sheet.

U. S. Department of Agriculture in cooperation with Frank Smith

Day of Week: Monday Date July 10 1911

	KIND OF WORK. Include implements used, number of loads, etc.	FIELD.	MAN HOURS.	HORSE. NO.	HORSE. HOURS.
4.30					
5.00					
5.30	milking & care of Cows		1		
6.00	Breakfast				
6.30					
7.00	Take milk to depot		1½	1	1½
7.30					
8.00					
8.30					
9.00					
9.30	Plowing Corn	J	4	1	4
10.00					
10.30					
11.00					
11.30					
12.00					
12.30	Dinner				
1.00					
1.30					
2.00	Hauling hay	B			
2.30	working with Mr. Smith		4½	2	9
3.00					
3.30					
4.00	Hauled in 4 loads				
4.30	about 1 ton each				
5.00					
5.30	care of team		½		
6.00	Supper				
6.30	milking		½		
7.00					
7.30					
8.00					
WORKMAN	O. Neal	TOTAL HOURS	12		14½

REMARKS Need rain. Corn suffering

Hay short

REPORT O. K.

F. S.

8—34

From the foregoing record a summary may be made on a blank form like the following illustration. At the end of the month the column totals will show the number of man hours and horse hours of labor expended on each of the farm's crops or other industries.

MONTHLY WORK REPORT

MONTH OF June 1911

	Corn		Wheat		Oats		Potatoes		Hay								Horses		Cattle		Hogs		Misc.		Total	
	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse
1	5	10					20	40									1½		5½		1				33	50
2	10	20					10	20									2		8	2	2½		2	2	34½	44
3	9	18					9½	19									1½		6½		1		4½	1	32	38
Sunday 4																	3½		5½		1½			4	10½	4
5																										
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30																										
31																										
Total																										

239. Finding the Cost of Man Labor and Horse Labor per Hour. At the end of a month or other period of time, the total number of man hours and horse hours can be determined from the records. The cost per hour may be found by the method illustrated below:

Cost of man labor per year

Wages,	$525.00
Board,	100.00
Total,	$625.00

Hours worked during year, 2500.

$625.00 ÷ 2500 = $.25, labor cost per man hour.

Cost of a horse-hour is determined in a similar way.

Total value of 2 horses,	$600.00
Int. on $600.00 at 5%,	$30.00
Feed and expense,	550.00
Total cost per year,	$580.00

Total number of horse labor hours, 6500.

$580.00 ÷ 6500 = 8.9 cents, labor cost per horse hour.

After the labor cost records have been made, the profit or loss from the crop may be determined by the preparation of a table similar to the following:

ACCOUNT WITH A CROP OF CORN IN FIELD B (36.48 ACRES) ON AN IOWA FARM, 1910[1]

ITEMS OF STATEMENT	DATE	TOTAL			PER ACRE			Per bushel
		Man hours[2]	Horse hours[2]	Cost	Man hours[2]	Horse hours[2]	Cost	
Plowing, fall of 1909 (14-inch gang)	Mar. 25 to Apr. 2	85½	342	$46.48	2.34	9.38	$1.279	——
Disking	Apr. 7 to 29	90¼	861	49.28	2.47	9.89	1.351	——
Harrowing	Apr. 29 to May 4	25½	51	8.56	.70	1.40	.235	——
Planting (with planter)	Apr. 30 to May 5	30½	61	10.25	.84	1.67	.281	——
Harrowing (after planting)	May 10 to 14	33⅜	72½	11.86	.92	1.99	.325	——
Cultivating (first time)	May 27 to 30	58	116	19.49	1.59	3.18	.534	——
Cultivating (second time)	June 3 to 6	54½	109	18.31	1.49	2.99	.502	——
Cultivating (third time)	June 14 to 18	51½	103	17.31	1.41	2.82	.474	——
Cultivating (fourth time)	June 23 to July 5	57	114	19.15	1.56	3.12	.525	——
Picking seed corn	Sept. 27 to Oct. 7	59	——	7.44	1.62	——	.204	——
Husking (from standing stalks)	Nov. 2 to 22	305½	611	102.65	8.38	16.75	2.814	——
Total labor cost	——	851	1,940½	310.98	23.32	58.19	8.524	——
Manure charge	——	——	——	124.91	——	——	3.424	——
Seed, 5½ bushels, at $5	——	——	——	27.50	——	——	.754	——
General expense	——	——	——	18.24	——	——	.500	——
Equipment	——	——	——	23.27	——	——	.638	——
Taxes	——	——	——	25.58	——	——	.700	——
Interest (rent)	——	——	——	255.35	——	——	7.000	——
Total cost	——	——	——	785.78	——	——	21.54	——
Summary:								
Income[3]	——	——	——	1,127.36	——	——	30.90	$0.512
Cost	——	——	——	785.78	——	——	21.54	.357
Profit	——	——	——	331.58	——	——	9.36	.155

[1] Previous crop: Timothy for seed, 1909.

[2] Rates per hour: Man hours, 12.6 cents; horse hours, 10.5 cents.

[3] Yield: 2,200 bushels of grain, at 50 cents (average, 60.3 bushels per acre), $1,100; stalks, $27.36; total, $1,127.36.

The labor records show the number of horse hours and man hours spent on each crop. After determining the hourly rate, it is necessary to multiply as follows:

The number of man hours by the rate per man hour; the number of horse hours by the rate per horse hour; to determine the total labor cost of each.

The costs and returns of each crop may then be summarized, and the gain computed, as illustrated in the following table:

CORN CROP ON SOUTH 40

ITEM		40 ACRES	PER ACRE
Seed		$ 24.00	.60
Fertilizer		70.00	1.75
Taxes		28.30	.7075
Interest		450.00	11.25
Expense		40.00	1.00
Man labor, 991 hours at 11 cents		109.01	2.725
Horse labor, 2286 hours at 10 cents		228.60	5.715
Total cost		949.91	23.7475
Yield, 2552 bu. corn at $.48	$1224.96		
Stalks, 75¢ per acre	30.00		
Total returns		1254.96	31.374
		949.91	23.7475
Gain		305.05	7.6265

The illustrations in this section are reprinted by permission from Farmers' Bulletin 511, U. S. Department of Agriculture.

Written Work

1. Find the labor cost to be charged against the corn crop, in the production of which 136 man hours and 312 horse hours of labor were expended. The total number of hours of man labor for the year was 4825, and the annual cost was $868.50; total number of hours of horse labor, 4312; total cost of horse labor for the year, $388.08.

2. Use the following facts to prepare a table similar to the model above:

Corn Crop on 20 Acres

Yield: 10 bushels of seed corn worth $3.50 per bushel.
Corn crop harvested, 67 bushels per acre, worth 54 cents per bushel.
Stalks were estimated at a value of 60 cents per acre.

Costs: $3\frac{1}{2}$ bushels of seed at $4.50 per bushel.
Fertilizer, $38.00.
Taxes: The farm was worth $175.00 per acre, and was taxed on $\frac{1}{5}$ the real value at a rate of $1.65. The farm contained 160 acres, and the taxes were prorated among the various fields in proportion to their acreage.
Interest: 4% on the value of the land.
Expense: $31.25.
Labor: 560 man hours at 19 cents.
1348 horse hours at 9 cents.

APPENDIX

DENOMINATE NUMBERS

SURVEYORS' LONG MEASURE

7.92 inches	= 1 link (lk.)
25 links	= 1 rod
4 rods, or 100 links	= 1 chain (ch.)
80 chains	= 1 mile

SURVEYORS' SQUARE MEASURE

625 square links	= 1 square rod
10 square rods	= 1 square chain
16 square chains	= 1 acre
640 acres	= 1 square mile
36 square miles	= 1 township

The following units of measure are used by sailors:

6 feet	= 1 fathom (Used for measuring depths at sea.)
120 fathoms	= 1 cable length (Used for measuring depths at sea.)
About 1.15 statute miles	= 1 knot, or 1 nautical mile, or 6080.27 ft.

CIRCULAR OR ANGULAR MEASURE

60 seconds (60″)	= 1 minute (′)
60 minutes	= 1 degree (1°)
90 degrees	= 1 right angle
360 degrees	= 1 circumference

TROY WEIGHT

24 grains (gr.)	= 1 pennyweight (pwt. or dwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)

APOTHECARIES' WEIGHT

20 grains (gr.)	= 1 scruple (sc. or ℈)
3 scruples	= 1 dram (dr. or ʒ)
8 drams	= 1 ounce (oz. or ℥)
12 ounces	= 1 pound (lb.)

Apothecaries' weight is used by physicians and druggists. Troy weight is used in the measurement of precious metals.

COMPARISON OF TROY AND AVOIRDUPOIS WEIGHTS

1 pound Troy	= 5760 grains
1 pound avoirdupois	= 7000 grains
1 ounce Troy	= 437½ grains
1 ounce avoirdupois	= 480 grains

The term *carat* has two meanings:

In weighing precious stones, a carat usually means 3.2 Troy grains.

In expressing the purity of gold, 24 carats means pure gold; 18 carats means $\frac{18}{24}$ pure gold, and $\frac{6}{24}$ alloy.

PAPER MEASURE

24 sheets = 1 quire (qr.)
20 quires = 1 ream (rm.)

Although a ream contains 480 sheets, 500 sheets are usually sold as a ream.

STANDARD UNITS OF WEIGHT

1 barrel flour	weighs 196 pounds
1 barrel salt	weighs 280 pounds
1 barrel pork or beef	weighs 200 pounds
1 keg of nails	weighs 100 pounds

STANDARD BUSHELS IN MANY STATES

1 bushel shelled corn	weighs 56 pounds
1 bushel ear corn	weighs 70 pounds
1 bushel wheat	weighs 60 pounds
1 bushel barley	weighs 48 pounds
1 bushel rye	weighs 56 pounds
1 bushel oats	weighs 32 pounds

TABLE OF ABBREVIATIONS USED IN BUSINESS

(The singular form is commonly used for both the singular and plural.)

A. . . . acre	ans. . . . answer
acct. or a c. account	Apr. . . April
agt. . . agent	Aug. . . August
amt. . . amount	av. . . . average

TABLE OF ABBREVIATIONS USED IN BUSINESS — (*continued*)

bg. . . . bag; bags
bal. . . . balance
bbl. or brl. barrel
bdl. . . . bundle
bkt. . . . basket
bl. . . . bale
B/L . . bill of lading
bot. . . . bought
bu. . . . bushel
bx. . . . box
C . . . one hundred
cd. . . . cord; card
cg. . . . centigram
ch. . . . chain; chest
chg. . . charge
c.i.f. . . carriage and insurance free
ck. . . . check
cm. . . . centimeter
cml. . . commercial
c/o . . . care of
Co. . . . company; county
c.o.d. . . collect on delivery
coll. . . collection
com. . . commission
consg't . . consignment
cr. . . . creditor; credit; crate
cs. . . . case
csk. . . . cask
c., ct. . . cent; centime
cu. ft. . . cubic foot
cu. in. . . cubic inch
cu. yd. . . cubic yard
cwt. . . hundredweight
d. . . . pence
da. . . . day
Dec. . . December
dep't . . department
dft. . . . draft
disc. . . discount
do. . . . ditto (the same)
doz.; dz. . dozen; dozens
Dr. . . . debit; debtor; doctor
E. . . . East
ea. . . . each
e.g. . . . for example
e.o.e. . . errors and omissions excepted
etc. . . . and so forth
ex. . . . example; express
exch. . . exchange
exp. . . expense
f. . . . franc
far. . . . farthing
Feb. . . February
fir. . . . firkins
f.o.b. . . free on board
frt. . . . freight
ft. . . . foot
gal. . . . gallon
gi. . . . gill
gr. . . . grain
gro. . . . gross
gr. gro. . great gross
guar. . . guaranty; guarantee
hf. . . . half
hf. cht. . half chest
hhd. . . hogshead
hr. . . . hour
i.e. . . . that is
in. . . . inch; inches
ins. . . . insurance
inst. . . instant; the present month
int. . . . interest
I.; inv. . invoice
inv't . . inventory
Jan. . . January
kg. . . . keg
km. . . . kilometer
l. . . . link
lb. . . . pound
l.p. . . . list price
ltd. . . limited
M . . . one thousand
m. . . . mill; meter
Mar. . . March
mdse. . . merchandise

TABLE OF ABBREVIATIONS USED IN BUSINESS — (*continued*)

Messrs. . *Messieurs;* Gentlemen; Sirs
mi. . . . mile
min. . . minute
mo. . . . month
mortg. . . mortgage
Mr. . . . Mister
Mrs. . . Mistress
N. . . . North
no. . . . number
Nov. . . November
Oct. . . October
O.K. . . all correct
oz. . . . ounce
p. . . . page
pay't . . payment
pc. . . . piece
pd. . . . paid
per . . . by; by the
per cent . per centum; by the hundred
pfd. . . . preferred
pk. . . . peck; pecks
pkg. . . package
pp. . . . pages
pr. . . . pair
prox. . . . the following month
pt. . . . pint
pwt. . . . pennyweight
qr. . . . quire
qt. . . . quart
rd. . . . rod
rec'd. . . . received
rec't . . . receipt
rm. . . . ream
Rm. (or M.) Reichsmark; Mark
s. . . . shilling; shillings
S. . . . south; sales
sec. . . . second
sec'y . . secretary
Sept. . . September
set. . . . settlement
ship. . . shipment
shipt. . . shipped
sig. . . . signed; signature
sq. ch. . . square chain
sq. ft. . . square foot
sq. mi. . . square mile
sq. rd. . . square rod
sq. yd. . . square yard
stk. . . stock
sund. . . sundries
T. . . . ton
tb. . . . tub
Tp.; Twp. township; townships
tr.; trans. . transfer
treas. . . treasurer; treasury
ult. . . . last month
via. . . . by way of
viz. . . . namely; to wit
vol. . . . volume
wk. . . . week
wt. . . . weight; weigh
yd. . . . yard
yr. . . . year

TABLE OF SYMBOLS

a/c . account
a/s . account sales
+ . . addition
( ) . aggregation
& . . and
... . . and so on
@ . . at; each; to
c/o . care of
¢ . . cent
√ . . check mark; correct
° . . degree
÷ . . division
$ . . dollar; dollars
= . . equal; equals

TABLE OF SYMBOLS — (*continued*)

′	foot; feet; minutes	o/d	on demand
$^{1}, ^{2}, ^{3}$	fourths (written as exponents, thus, $3^{1} = 3\frac{1}{4}$)	%	per cent
		£	pounds sterling
>	greater than	:	ratio
C	hundred	∵	since
″	inch; inches; seconds	−	subtraction
<	less than	∴	therefore
×	multiplication	M	thousand
#	if written before figures, means number; if written after figures, means pounds		

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